Plasticity is the property of the solid body to deform under applied external force and to possess permanent or temporal residual deformation after applied load is removed.

Main feature of plasticity: $\sigma \sim \varepsilon$

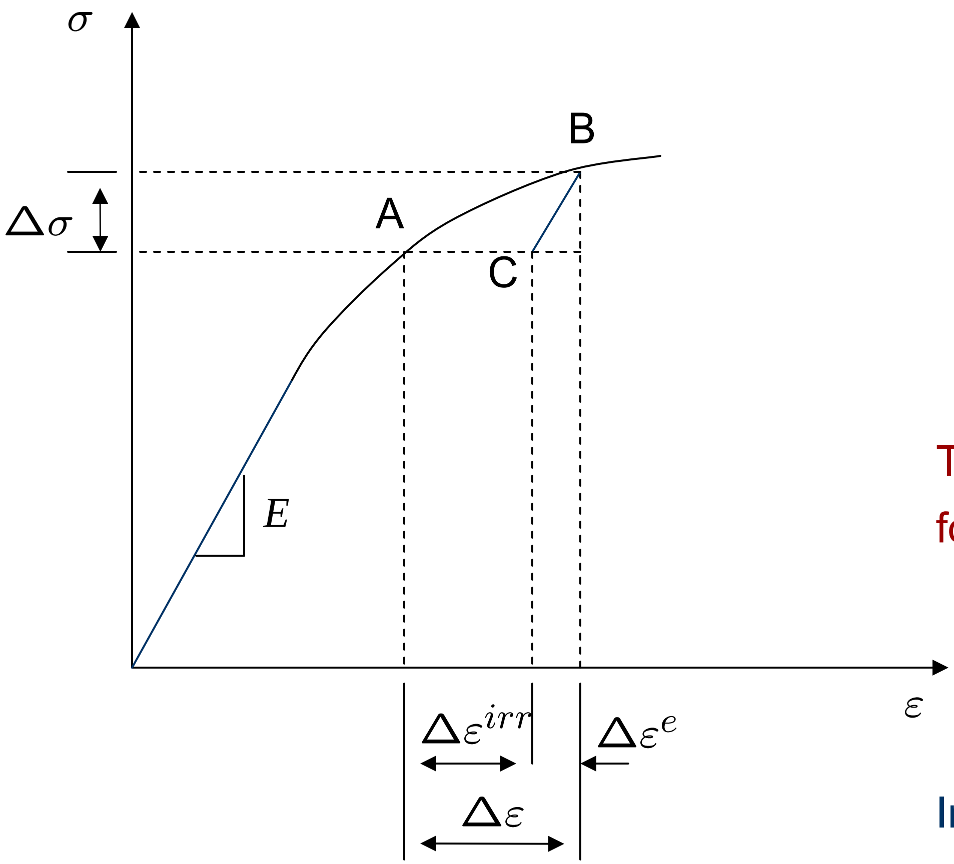
is not uniquely determined by the current state.

Elastic state of a solid body is a state at which an independent of time uniquely determined relationship between stresses and strains exists for any given temperature.

Plastic state of a solid body is a state at which for a given temperature the relationship between stresses and strains at each moment of time becomes uniquely determined if at least one (or all) preceding stress-strain state and the corresponding to it temperature are known.

Otherwise $\sigma \sim \varepsilon$ is not determined.

1D ⟷ 3D



uniaxial test

- Reversible elastic deformation
- Irreversible, inelastic deformation

Term plastic deformation will be used for inelastic time – independent strain

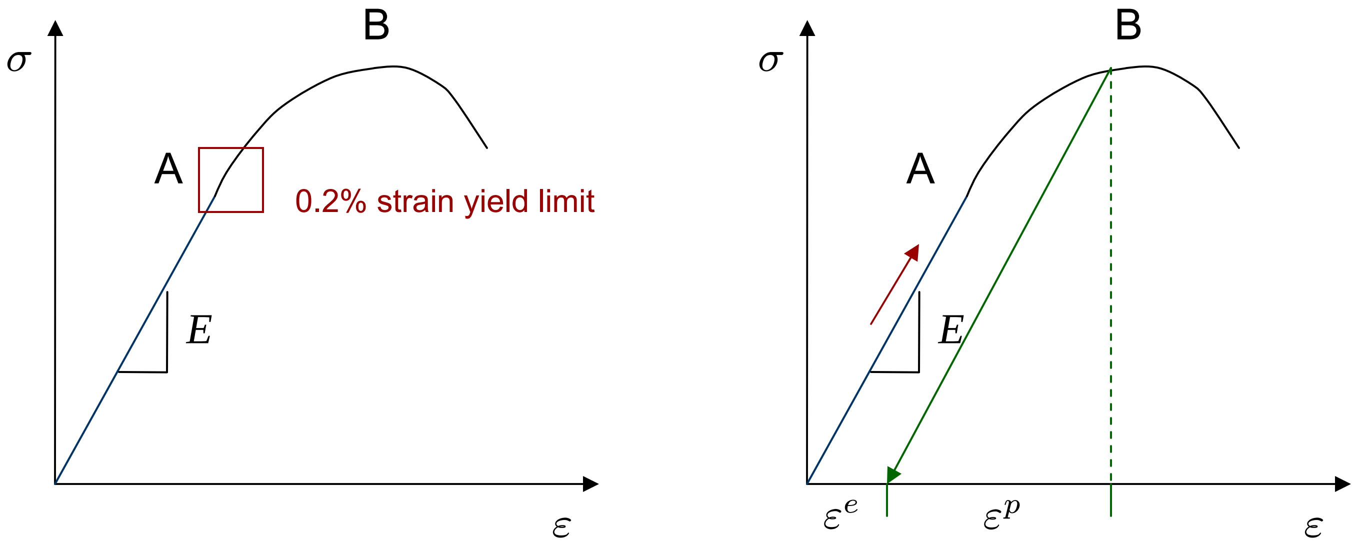
Inelastic deformation may be:

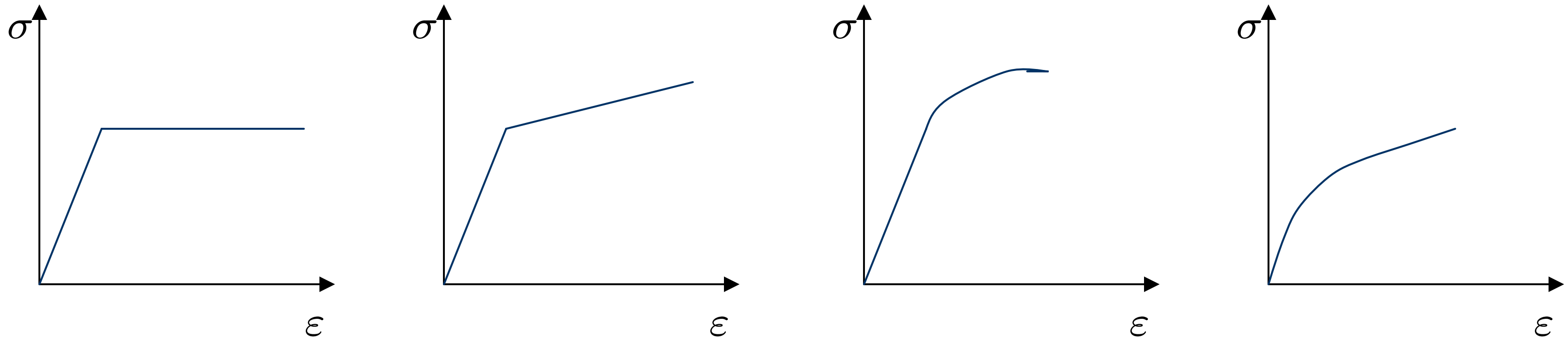
– creep (time dependent);

– viscoplastic (rate dependent)

1D ⟷ 3D

uniaxial test

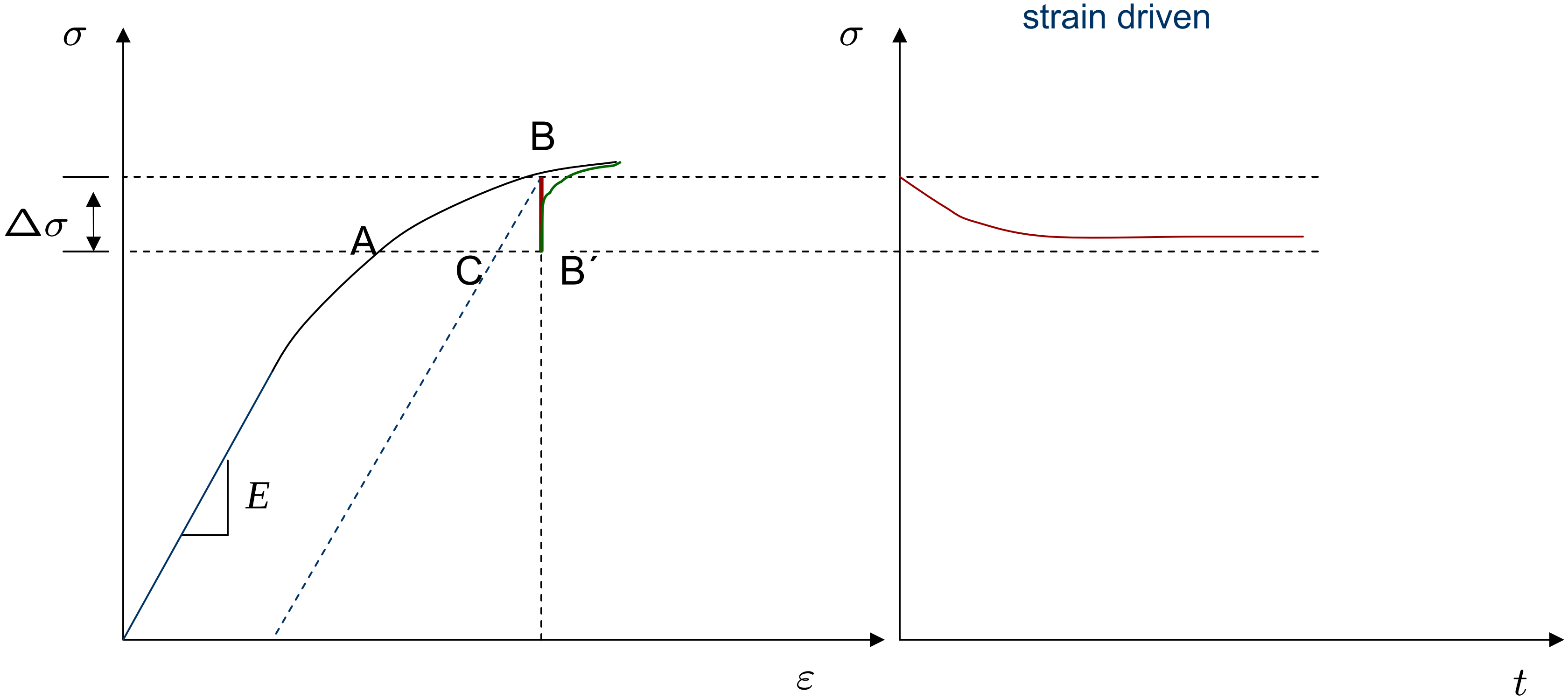




1D ⟷ 3D

uniaxial test – relaxation

strain driven



- Reversible elastic deformation
- Irreversible, inelastic deformation

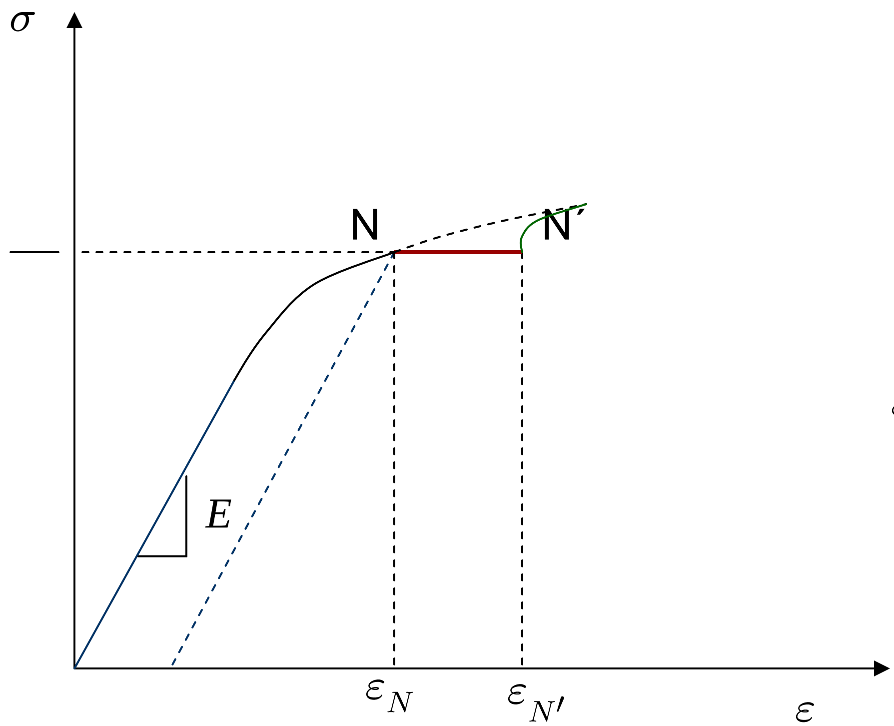
BB´- path of keeping the deformation constant

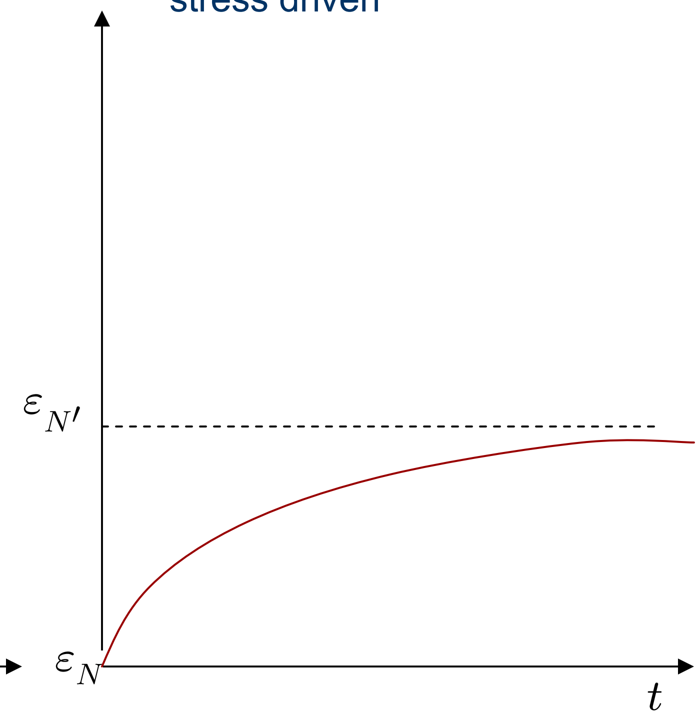
more pronounced at high temperature

1D ⟷ 3D

uniaxial test – creep

stress driven





- Reversible elastic deformation
- Irreversible, inelastic deformation

NN´- path of keeping the stress constant

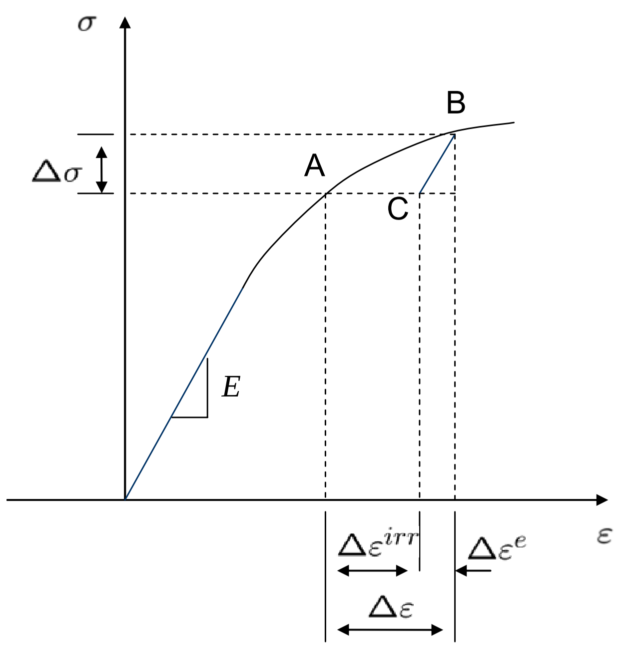
- creep

more pronounced at high temperature

Elasto-plastic problem:

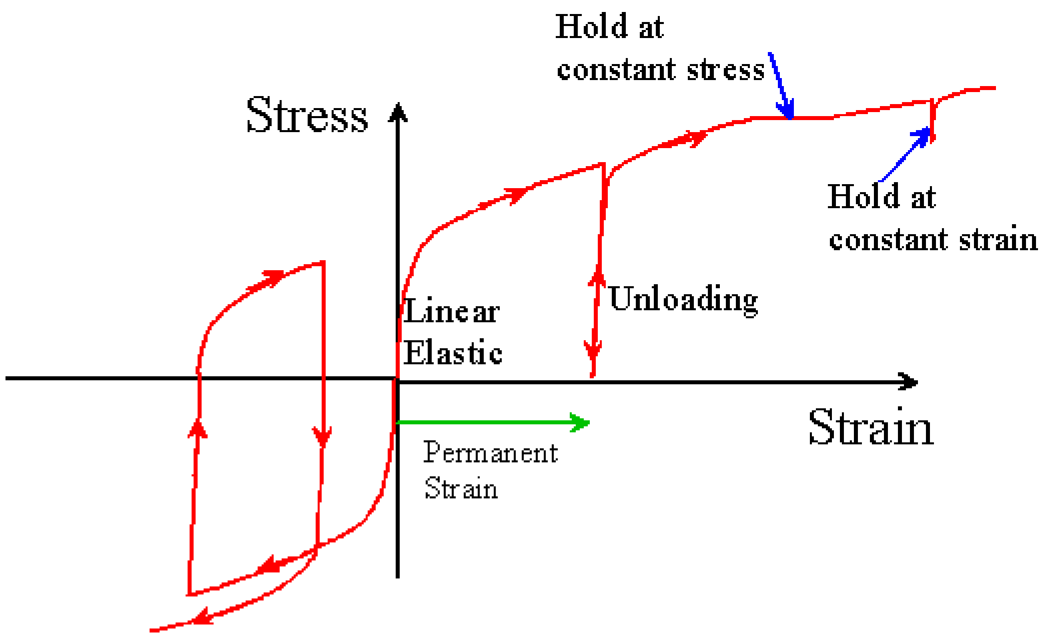
Given external forces (static or dynamic) determine

- deformations
- residual deformation after complete or partial unloading
- changes in the material properties due to the plastic deformation
- deformation path during reloading
- loading at failure



- Reversible elastic deformation
- Irreversible, inelastic deformation

1D ⟷ 3D



typical tension/compression test on an annealed, ductile, polycrystalline metal specimen (e.g. copper or Al)

Key ideas in modelling metal plasticity

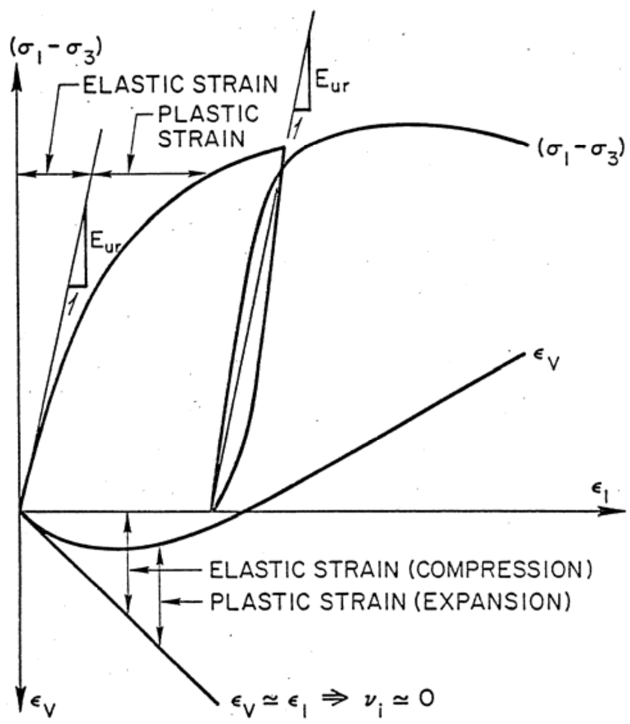
1. The decomposition of strain into elastic and plastic parts
2. Yield criteria: which predict whether the solid responds elastically or plastically
3. Strain hardening rules, which control the shape of the stress-strain curve in the plastic regime;
4. The plastic flow rule, which determines the relationship between stress and plastic strain under multi-axial loading;
5. The elastic unloading criterion, which models the irreversible behaviour

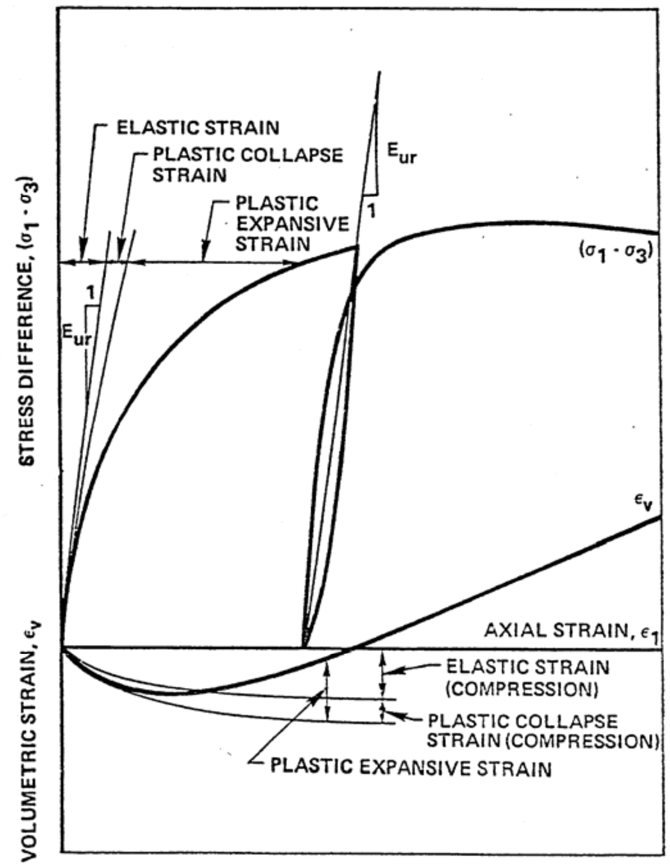
1D ⟷ 3D

conventional triaxial test (soil, concrete)

compression is positive

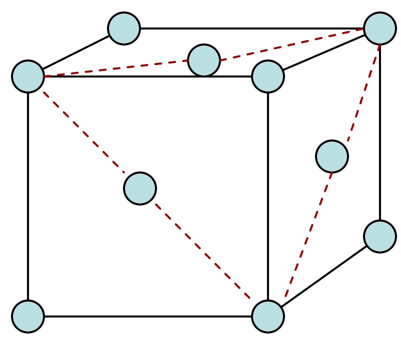
$$\sigma_1 > \sigma_2 = \sigma_3$$





Plastic deformation (also failure) for metals is considered as slip or dislocation of crystals.

- Dislocation Theory of Plasticity - Sliding theory (solid state physics plastic theory) based on the dislocation theory that regards plastic deformation of each mono-crystal to occur in a plane and there in a direction of most dense packing of molecules.



shortcomings:

all active sliding systems contribute to the cumulative irreversible deformation and this contribution is additive – usage of superposition principle in nonlinear effects (as plasticity is) is shady.

it is assumed the stress state of each crystalline to be the same as if the body is macro-homogeneous and this can lead to displacements discontinuities on the crystal interface boundaries.

Plastic deformation (also failure) for metals is considered as slip or dislocation in crystals.

$$\Downarrow$$

- the plastic deformation is associated with solely the shear deformation

  no volume change occurs due to plastic deformation

$$\varepsilon_{ij}^{p}\,\delta_{ij} = 0$$

$$p = K\varepsilon_v$$ $\rightarrow$ elastic law applies to the volumetric strain

- plastic behaviour in tension and compression is almost identical (von Mises plasticity)

Historically the plasticity theory has been developed in conjunction with this metal behaviour

metals, alloys	rocks, soils, ceramics, powders, concrete
polycrystalline microstructure	porous materials
plastic mechanisms: • sliding (slip or dislocation)	plastic mechanisms +: • collapse • development of micro-cracks
Shear plastic deformation volumetric strain is elastic	plastic volumetric deformation

yield limits

$$\sigma_t^y = \sigma_c^y \qquad\qquad \sigma_t^y \neq \sigma_c^y$$

modification of metal plasticity to model rock, soil and concrete inelastic behaviour

↓

- separation to deviatoric and volumetric paths – deviatoric reproduces metal plasticity
- modified with the adoption of the third stress/strain invariant

Total stress – total strain relations

Hencky, Nadai deformation theory (1920 – 1926 –>)

– hardening plasticity invoked by the application of aluminium alloys and related research activity at the beginning of last century

$$s_{ij} = 2\,G_s\,(J_2)\;e_{ij} \quad \text{if} \quad J_2 = \tilde{J}_2$$

$$ds_{ij} = 2\,G\,d\,e_{ij} \quad \text{if} \quad J_2 < \tilde{J}_2$$

$$p = K\varepsilon_v \quad \text{everywhere}$$

$$\Downarrow$$

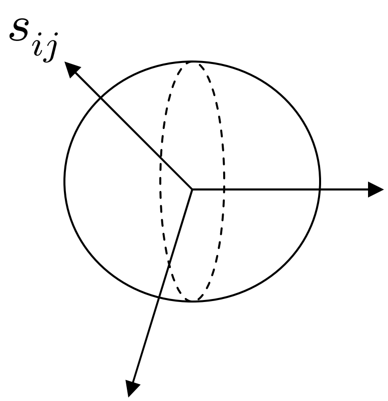
This is a particular linear with respect to the tensors kind of elastic theory

+

effect of unloading

Total stress – total strain relations

Hencky, Nadai deformation theory (1920 – 1926 –>)



$$J_2 = \tilde{J}_2 = const$$

a hypersphere called yield surface
and/ or loading surface

<u>Example (fracture mechanics, fretting):</u>

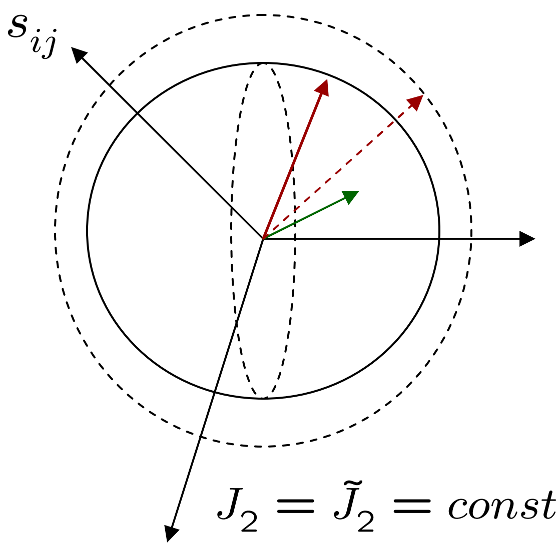
Ramberg – Osgood deformation theory. In uniaxial case it reads:

$$E\,\varepsilon = \sigma + \alpha \left(\frac{|\sigma|}{\sigma_0}\right)^{n-1} \sigma$$

power law hardening

linear

$$E\,\varepsilon = (1+\nu)\,\mathbf{S} - (1-2\nu)p\,\mathbf{I} + \frac{3}{2}\alpha\left(\frac{q}{\sigma^0}\right)^{n-1}\mathbf{S},$$



Total stress – total strain relation (+ and -)

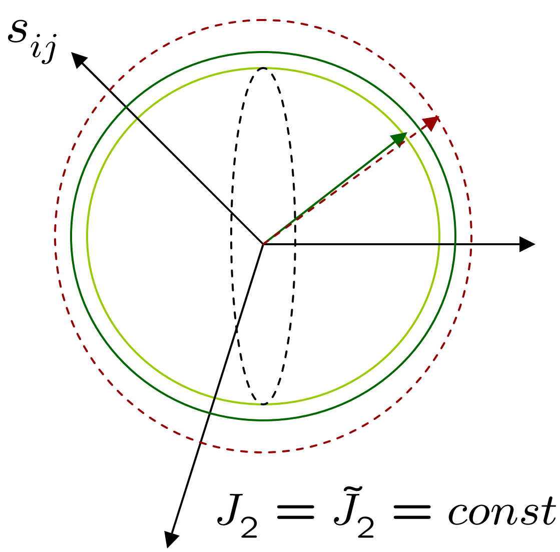
(+) simplicity and good prediction ability in cases when the loading is not much deviating from the radial path.

Ilyushin theorem: a deformation theory of plasticity is entirely adequate when the loading is simple; that is, when 1. all the applied forces grow in proportion to a single parameter,
2. the material is incompressible, $\varepsilon_v = 0$, and
3. $G_s(J_2) = A\,J_2^{\kappa}$

(-) The increment of the deformation is finite for infinitely close stress trajectories *

Ilyushin (1949) showed how a general plasticity theory for any complex loading may be constructed by successively adding quantities of the nature of correction terms to the deformation theory. All of the theories of plasticity so far suggested for the complex loading condition are shown to be special cases of this general theory.

→ If the loading is proportional and monotonically increasing then power law hardening deformation plasticity and incremental plasticity are essentially equivalent.



Active process:

$$ds_{ij} = 2\,G_s\,(J_2)\;de_{ij} + 2\,G_s'\,(J_2)\;e_{ij}\,dJ_2$$

Passive process:

$$ds_{ij} = 2\,G\,de_{ij}$$

If now

$dJ_2 \rightarrow 0$	neutral process

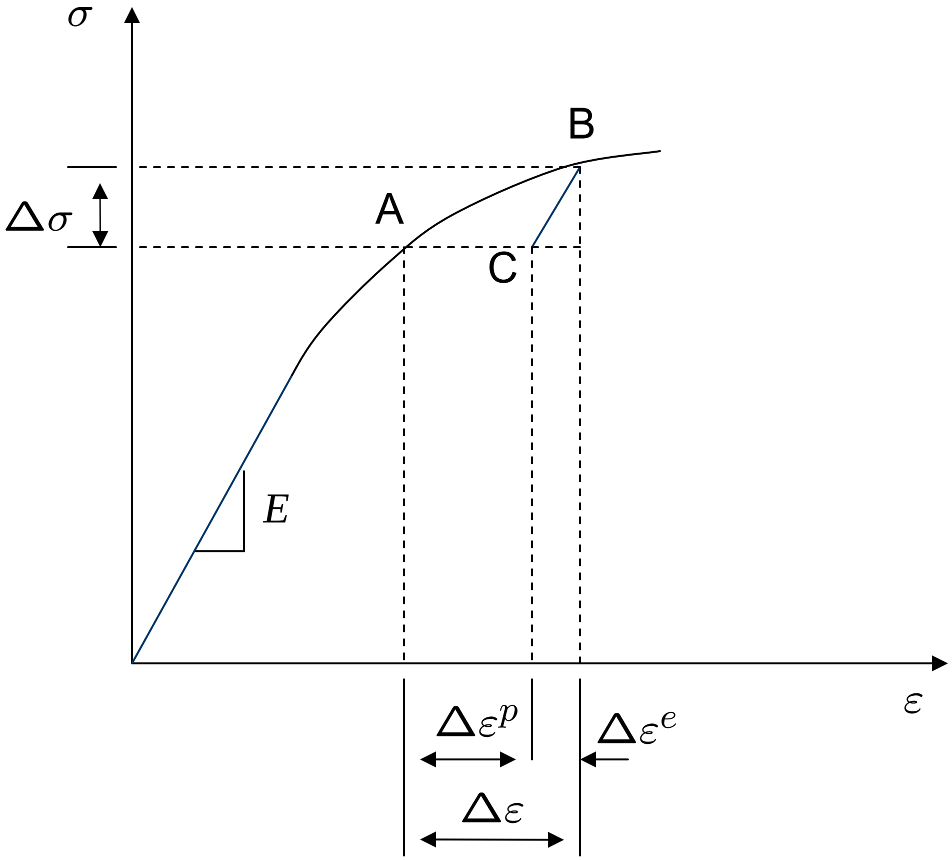
$$\Downarrow$$

$$\frac{(de_{ij})_{active}}{(de_{ij})_{passive}} = \frac{G}{G_s\,(J_2)}$$

*only active loading is implemented in numerical procedures

Consequently invoked

- Prager‘s consistency condition: two loading paths anyhow close to the neutral path must lead to equal resulting deformations.
- Prager‘s flow theory of plasticity



Plastic and elastic strains are additive:

$$\Delta\varepsilon = \Delta\varepsilon^e + \Delta\varepsilon^p$$

$$d\varepsilon = d\varepsilon^e + d\varepsilon^p$$

$$\dot{\varepsilon} = \dot{\varepsilon}^e + \dot{\varepsilon}^p$$

$$d\varepsilon^e = \frac{1}{E} d\sigma$$

$$d\varepsilon_{ij} = d\varepsilon_{ij}^e + d\varepsilon_{ij}^p$$

$$d\sigma_{ij} = C_{ijkl}^e \, d\varepsilon_{kl}^e$$

$$d\sigma_{ij} = C_{ijkl}^e \left(d\varepsilon_{kl} - d\varepsilon_{kl}^p \right)$$

The strain additivity does not necessarily hold and has to be considered as an assumption

Term “flow plasticity” comes from hydrodynamical analogy.
More relevant terms are **incremental plasticity** or **differential plasticity** theories,
Theories of plastic flow (Saint Venant-Lévy-Mises, Prandtl-Reuss, Prager)

Prager theory of plastic flow: consistency condition + the following assumptions ->

1. the increment of the strain deviator is fully determined by the stress deviator and its increment;
2. the relation is **linear** regarding de_{ij} and ds_{ij} in both elastic and plastic cases;
3. the current yield (limit) surfaces are hyperspheres as in deformation plasticity theory.

$$\Downarrow$$

Loading (additional charge/discharge) from plastic case + 1 to 3 yield:

$$de_{ij} = \begin{cases} A_{ijkl}\, ds_{kl} & dJ_2 \geq 0 \\ ds_{ij}/2G & dJ_2 \leq 0 \end{cases}$$