MATHEMATICAL MODELING OF THE VISCO-ELASTIC PROPERTIES OF HUMAN ABDOMINAL FASCIA

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ABSTRACT. Mechanical behaviour of umbilical and inguinal human abdominal fascia was studied. Mathematical modeling of the visco-elastic properties of investigated fasciae was accomplished applying the non-linear theory of Maxwell-Gurevich-Rabinovich (MGR). The results show that the non-linear theory describes satisfactory the experimental data.

KEY WORDS: mathematical modeling, abdominal fascia, visco-elastic properties

1. Introduction

Abdominal fascia is a dense connective tissue which is a part of the abdominal wall. Like most soft biological tissues the abdominal fascia has to be considered as a visco-elastic solid.

The most popular theory used for modeling the visco-elastic behaviour of soft biological tissues is that proposed by Fung [1]. Fung noted that the stress-strain relationships of soft biological tissues were insensitive to strain rate. The quasi-linear visco-elastic theory of Fung was applied to relaxation process of abdominal fascia [2]. It was shown however that the behaviour of same fascia samples could not be modeled with Fung's theory.

The description of the mechanical behaviour of the fascia requires applying of theory, which can account for the nonlinear, time dependant stress-strain characteristics of the fascia. Such a theory is proposed by Maxwell-Gurevich-

Rabinovich (MGR) [3]. It was developed to describe the deformation and relaxation process in polymers, but was successfully applied for description of the relaxation behaviour of soft biological tissue like the human small intestine [4].

The aim of this work is to proposed a model of the visco-elastic behaviuor of the human abdominal fascia using the nonlinear Maxwell-Gurevich-Rabinovich (MGR) theory.

2. Materials and methods

The fascia specimens were extracted from the posterior wall of inguinal canal and from the umbilical region. The specimens with dimensions between (10x40) - (10x70) mm were tested. The samples were oriented parallel to the collagen fibers (direction L1) and perpendicular to it (direction L2). Uniaxial relaxation tests on 19 flat specimens from umbilical fascia and 19 samples from transversalis fascia in longitudinal and transversal direction were performed. The investigation was done using computer equipped testing machine FU1000/E. The experiments were performed at 1.26 mm/sec rate of elongation. The initial length of the specimens was measured after preconditioning of the specimens. Strips were elongated from 2% to 15% of their initial length [5].

The experimental data were represented as stress-time relationships. From the obtained experimental load-elongation curves the Lagrangian stress σ (tensile force per unit undeformed cross-sectional area) was calculated.

According to the non-linear theory of Maxwell - Gurevich- Rabinovich the total deformation of the samples ε is a sum of the elastic deformation e and the viscous deformation ε^*.

(2.1) $$\varepsilon = e + \varepsilon^*$$

The one-dimensional stress relaxation process is governed by the following equation:

(2.2) $$\frac{d\sigma}{dt} = -\frac{E}{\eta_0^*} f^* \exp\left\{\frac{\frac{1}{3}\gamma^* \sigma + \left|f^*\right|}{m^*}\right\}$$

where η_0^* is the initial coefficient of viscosity, m^* is the parameter, determining the influence of the strain rate on the stress and γ^* is the volume deformation (compressibility coefficient). The function f^* is defined as:

(2.3) $$f = \sigma - E_\infty \varepsilon^* = (1 + \frac{E_\infty}{E})\sigma - E_\infty \varepsilon_0$$

Here ε_0 is the initial step-wise deformation of the sample, E is the local elasticity modulus at the beginning of relaxation process (t=0), E_∞ is the elastic

modulus when the relaxation process is completed. The solution of the differential equation (2.2) is :

$$(2.4) \qquad t = \frac{\overset{*}{\eta_0}}{E + E_\infty}\left[- E_i(-\xi^*) + E_i(-\xi_0^*)\right]$$

where E_i is the exponential integral function and the values of the argument ξ^* are calculated as:

$$(2.5) \qquad \xi^* = \frac{f^*}{m^*} = \left|(1 + \frac{E_\infty}{E})\sigma - E_\infty\varepsilon_0\right| / m^*$$

3. Experimental results

The non-linear theory of Maxwell-Gurevich-Rabinovich was applied for 6 specimens from umbilical fascia (U)F and 8 samples from fascia transversalis (FT). The theory describes satisfactory the experimental data [5]. The calculated error between theoretical and experimental results varied was in the range 0.402%-0.877% for FT and 0.164%- 0.725% for UF. The results are presented in Figs. 1-2.

Figure 1. compares the experimental and theoretical results for FT (experiment 185N1 and 65N2b) and UF (experiment 65N5 and 65N2c) cut in L1 and L2 directions at initial deformation 5.5%. It is evident that the agreement of the results is better in the first part of relaxation process up to 60-80 sec. Figure 2 presents the calculated time courses of the apparent viscosity $\eta^*(t)$ and viscous deformation $\varepsilon^*(t)$ for the same relaxed samples shown in Figs.1. One can see that the maximum values of viscosity parameter η_0^*, varies in wide interval between 250-3500 [MPa.sec]. The values of η_0^* in longitudinal direction for both fasciae are smaller than the those in transverse direction. The maximum values of viscous deformation ε^* are in the interval 1.7% - 2.5%. The viscous deformation ε^*(t) sharply increases in first part of relaxation process and than tends to some asymptotic value.

Comparing the influence of the orthotropy of the material on the asymptotic values of the $\varepsilon^*(t)$ and η^*(t), Fig.2 clearly shows that $\varepsilon^*(t)$ is almost insensitive to directions L1 and L2. This evidence, however, needs additional investigations.
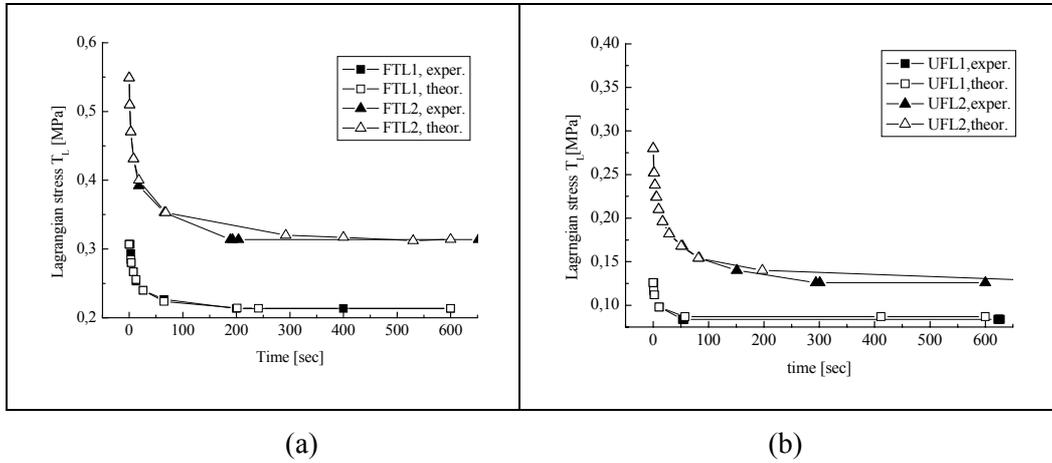
(a)   (b)

Fig. 1. Comparison of experimental and theoretical results from both investigated direction . (a) – Results for FT and (b) – results for UF.
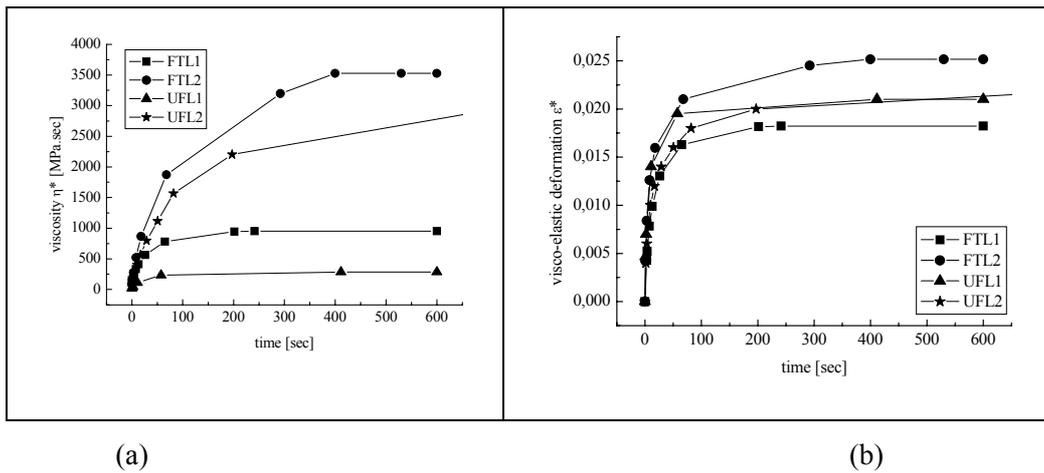

(a)   (b)

Fig. 2. The time course of viscosity parameter η^* - Figs.2 (a) and viscous

deformation ε^* - Figs.2 (b) for both fasciae, directions L1 and L2.

4. Conclusion

The obtained results show that, based on simple one-dimensional relaxation experiments, the MGR theory gives an opportunity to elucidates the basic characteristics of the visco-elastic behaviour like time dependence of the apparent viscosity and the viscous deformation. The non-linear MGR theory describes satisfactory the experimental data. The initial viscosity parameter η_0^*, varies in wide

interval not only among samples from different patients but also between samples from one patient. The values of η_0^*, correlate with the values of the initial stress [5]. The time courses of the apparent viscosity η(t) and viscous deformation ε(t) give the possibility to assess the contribution of the viscous effects to the short and long-term mechanical behavior of the human abdominal fascia.

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