FEM APPLIED IN HYDRO-MECHANICAL COUPLED ANALYSIS OF A SLOPE

A. Yanakieva, M. Datcheva, R. Iankov, F. Collin, A. Baltov

1. Institute of Mechanics, Bulgarian Academy of Sciences, Acad. G. Bontchev, bl.4, BG1113 Sofia
2. University of Liege, Belgium

aniyanakieva@imbm.bas.bg

1. Introduction
Landslides can be attributed to a number of factors, such as geologic features, topography, vegetation, weather, or combinations of these factors. Rock and soil mass is involved in tectonic or other natural processes that result in appearance and development of different kind of discontinuities such as faults, cracks and breaks filled with water or softer/harder minerals. The existence of such discontinuities is one of the essential failure sources in landslides. When water is present in or near the discontinuities, as well as when the water is in the area of contact between two or more different soil layers, decrease of soil internal resistance is accounted for by means of shear strength decrease on the potential failure surface. The present study assumes that the cause of the fracture processes is the development of stress and/or strain localization bands. Recently, the Finite Element Method (FEM) provides a more powerful alternative to traditional methods in assessing stability of non-reinforced slopes and embankments. This paper presents an example of application of a coupled hydro-mechanical finite element model to simulate the stress and strain state in a slope and to investigate the influence of the existence of a fault on the slope behavior. A special finite element is used to model the influence of water pressure that is applied into the fault. To elucidate the impact of the fault water pressure variation on the potential failure hereafter two numerical simulations are performed. One of them considers the application of hydrostatic pressure on part of the slope surface which is near the fault, and the other one does not consider the application of hydrostatic pressure. The change of the slope mechanical behavior considering two or more soil layers and more than one faults, will be an object of further studies.

2. Coupled hydro-mechanical model
A large strain finite element formulation of the mechanical problem is used to describe the coupled hydro-mechanical behavior of the slope. In particular, the kinematics of solid continuum is described using Lagrangian formulation. To express the equilibrium update Lagrangian formulation is applied in the current configuration. In terms of the FEM approach, this means that the reference configuration evolves at each time step.

The Cauchy stress tensor $\sigma_{ij}^{total}$ is chosen as a stress measure. The general relation for the static equilibrium of an elementary volume is given by:

$$\sigma_{ij}^{total} + \rho b_j = 0, \quad (i, j = 1,2,3),$$

where $\rho$ is the unit volume mass and $\rho b_i$ is the mass force.

Terzaghi effective stress hypothesis is assumed and the effective stress $\sigma_{ij}$ is defined as:

$$\sigma_{ij} = \sigma_{ij}^{total} - p_w \delta_{ij} \quad \text{if} \quad p_w \geq 0 \quad \text{and} \quad \sigma_{ij} = \sigma_{ij}^{total} \quad \text{if} \quad p_w < 0$$

where $p_w$ is the water pressure.

The strain rate tensor is a sum of elastic strain rate, $\dot{\varepsilon}_{ij}^{e}$ and plastic strain rate, $\dot{\varepsilon}_{ij}^{p}$:

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{e} + \dot{\varepsilon}_{ij}^{p},$$

The elastic part of the strain rate is related to an objective rate of the stress (i.e. Jaumann derivative) through a linear elastic law and the plastic part obeys a plastic flow rule defined for each of the plastic mechanisms $\alpha$ in the following way,[1], [4]:

$$\dot{\varepsilon}_{ij}^{p} = \dot{\lambda}_{ij}^{p} \frac{\partial g_{\alpha}}{\partial \sigma_{ij}}$$

412
while $\lambda^p$ is the plastic multiplier; $g_\alpha$ is a potential function which may be different for different plastic mechanisms $\alpha$. The corresponding plastic flow rule reads $f_\alpha = 0$ and is defined by so called discontinuous plastic flow function in the stress space, [1]. We assume in the present work that $\alpha = 1$. Then, the plastic flow function is given as:

$$f_i = II_\sigma + m \left( I_\sigma - \frac{3c}{\tan \phi_c} \right) = 0,$$

(5)

where $I_\sigma = \sigma^i_j$, is the first stress invariant and $II_\sigma = \frac{1}{2} \hat{\sigma}^i_i \hat{\sigma}^i_i$, is the second stress invariant,

$$\hat{\sigma}^i_i = \sigma^i_j - \frac{1}{3} \delta^i_i \delta^j_j,$$

is a stress deviator, $c$ is cohesion, $\phi_c$ is the angle of friction and $m$ is a factor depending on the model. In the case of Drucker-Prager model $m = \frac{2 \sin \phi_c}{\sqrt{3} (3 - \sin \phi_c)}$, and for Van-Eeckelen model $m = a(1 + b \sin 3\beta)^n$, where $a$ and $b$ are functions of $\phi_c$, $\phi_c$ and $n$ is a Van-Eeckelen parameter, $\beta$ is the Lode angle which depends on the third invariant of the stress deviator.

The non-associate plastic flow rule is introduced using dilatancy angle $\psi$, and the plastic potential expression reads:

$$g_1 = II_\sigma + m^t \left( I_\sigma - \frac{3c}{\tan \psi} \right).$$

(6)

The equivalent plastic strain rate is given by

$$\dot{\varepsilon}^p_{eq} = \frac{2}{3} \hat{\varepsilon}^p_{ij} \hat{\varepsilon}^p_{ij} = \lambda^p \left( \frac{2}{3} \frac{\partial g_1}{\partial \sigma^i_j} \frac{\partial g_1}{\partial \sigma^i_j} - \frac{1}{3} \frac{\partial g_1}{\partial \sigma^i_i} \frac{\partial g_1}{\partial \sigma^i_i} \right),$$

(7)

where $\hat{\varepsilon}^p_{ij}$ is the plastic strain rate deviator. Material hardening/softening is described based on the evolution of cohesion and friction angle as functions of the equivalent plastic strain $\varepsilon^p_{eq} = \int_0^t \dot{\varepsilon}^p_{eq} dt$.

Fluid flow is described by the generalized Darcy law

$$f_{w}(u) = -k_{int} k_{rw} \left[ p_{w,j} + g \rho_w y_j \right]$$

(8)

where $f_w$ is the Darcy fluid flux; $k_{int}$ is the intrinsic permeability; $\mu_w$ is the dynamic viscosity coefficient; $g$ is the gravity coefficient; $\rho_w$ is the water density; $k_{rw}$ is the relative permeability, and $y$ is the up down coordinate.

### 3. FEM model and numerical examples

Two examples for a slope with a fault are studied. In the first one it is considered a hydrostatic pressure applied near the fault (model CON4), while the second one is without such loading condition (model VER20).

The geometry of the boundary value problem and the space discretization are shown in Fig. 1a. The coupled hydro-mechanical model described in the Section 2 is used for the slope material description. It is assumed that the material is isotropic.

The mechanical and hydraulic boundary conditions applied are depicted in the Fig. 1b and are that follows: fixed horizontal displacements ($u_x$) of the left and right model boundaries; fixed vertical displacements ($u_y$) on the bottom boundary; for model CON4 only hydrostatic pressure is applied on part of the slope surface attaining a value of 1E5 kPa for a prescribed period of time.

Dead weight load acts within the whole volume.

Numerical solution of the problem is obtained by the FE code Lagamine – ULg, specially its soil version, a tool designed to handle coupled problems in geological media, [4]. Material and model parameters used in code Lagamine
are given in the Tables 1 and 2; parameters of the mechanical model are given in Table 1; and hydraulic properties are given in Table 2.

![Slope geometry](image1)

![Boundary conditions](image2)

Fig. 1a Slope geometry – dimensions: $L_1$=64 m; $L_2$=20 m; $L_3$=20 m; $H_1$=40 m $H_2$=20 m; A=8.83 m.

Fig. 1b Boundary conditions

<table>
<thead>
<tr>
<th>parameter</th>
<th>Yong elastic modulus $E$ (MPa)</th>
<th>Poisson ratio $\nu$</th>
<th>Dilatancy angle (for compressive paths) $\psi_e/\psi_c$ (°)</th>
<th>Specific mass $\rho$ (kg/m$^3$)</th>
<th>Initial/Final Coulomb’s angle (for compressive paths) $\varphi_\alpha/\varphi_{cf}$ (°)</th>
<th>Initial/Final value of cohesion $c$ (MPa)</th>
<th>Only if there is hardening/softening $B_p/B_c$</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>200</td>
<td>0.25</td>
<td>10/10</td>
<td>2650</td>
<td>35/35</td>
<td>1.0/1.0</td>
<td>0/0</td>
</tr>
</tbody>
</table>

Table 2

<table>
<thead>
<tr>
<th>parameter</th>
<th>Soil permeability $k_{int}$ (m$^2$)</th>
<th>Specific mass of the fluid $\rho$ (kg/m$^3$)</th>
<th>Soil porosity $n_0$</th>
<th>Fluid compressibility coefficient $1/k_w$ (Pa$^{-1}$)</th>
<th>fluid dynamic viscosity $\mu_w$ (Pa.s)</th>
<th>CSR1 (Pa)</th>
<th>CSR2 (Pa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>1E-13</td>
<td>1000</td>
<td>0.5</td>
<td>3.33E-10</td>
<td>1E-3</td>
<td>1E4</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>parameter</th>
<th>CSR3</th>
<th>SRES</th>
<th>SRES</th>
<th>$S_{field}$</th>
<th>CKW1</th>
<th>CKW2</th>
</tr>
</thead>
<tbody>
<tr>
<td>value</td>
<td>1</td>
<td>0.1</td>
<td>1</td>
<td>4</td>
<td>4</td>
<td></td>
</tr>
</tbody>
</table>

The fault is numerically modeled by means of a special finite element, which allows introducing water pressure into the fault as a third unknown in the element nodes. This type of FE is incorporated into the FE code LAGAMINE and is described in [2]. The applicability of this finite element is verified in [3].

In both example cases CON4 and VER20 it is assumed that the water-level reaches height of $H_2$. Since there is a fixed water level, the slope part that is above that level is considered to be in unsaturated condition, and the slope part below that level – in water saturated condition. Hence, a function of soil-water interaction, $S_w(s)$, and a function determining relative permeability, $k_{rw}(S_w)$, have to be defined. For the model with key ISRW=8, the expression for $S_w(s)$ reads, [2]:

$$S_w(s) = \frac{CSR3}{\pi} \arctan \left( \frac{s + CSR2}{CSR1} \right) + \frac{CSRs}{2},$$

where $s = p_a - p_w$, $p_a$ is pore air pressure. In our case the system is opened to the air to flow out and therefore it can be taken that $p_a = 0$. For the model with key IKW=4, the expression for the relative permeability $k_{rw}(S_w)$ has the form [2]:

$$k_{rw} = \begin{cases} 
(S_w - S_{res})^{CKW1}_{S_{field} - S_{res}}^{CKW2} & \text{if } S_w \geq S_{res}; \\
S_{rw} & \text{if } S_w < S_{res}.
\end{cases}$$

\[9\]
The body of the slope is modeled by means of a 2D solid hydro-mechanical coupled FE. The initial stresses in the slope are obtained using an excavation technique. The simulation starts for a model with rectangular geometry. After several steps, the requested number of finite elements is removed that finally the model geometry attains the slope shape. Then, the calculation proceeds with applying loading conditions on the slope boundary that is in our case prescribing hydraulic head on the part of the slope next to the fault. The applied pore water pressure as a function of time is shown in Fig. 2.

Fig. 2 Applied pore water pressure as a function of time

The influence of pore water pressure on the evolution of equivalent strain has been further studied. The evolution of the equivalent strain for case model CON4 is presented in Fig. 3.1, 3.2, 3.3, 3.4. The results are given at four different times, namely $t_1=1E4$, $t_2=7E4$, $t_3=9E4$ and $t_4=11E4$ seconds. The evolution corresponds to the different water pressure applied due to the water injection along the slope boundary, just above the fault. The results show an increase of the maximal value of the equivalent strain at two characteristic points of the area under consideration – the slope toe and the fault tip. For a certain value of the pressure applied, the line connecting those points is possibly the most probable one along which a strain localization line is expected to occur. This is an area where fracture can develop. For a comparison in Fig. 3.5, 3.6, 3.7, 3.8 at the same time steps the evolution of equivalent strain is depicted for the model example VER20 when there is no application of hydrostatic pressure. Comparing figures 3.1-3.4 and 3.5-3.8 it is evident that the injection of water is the reason for the development of a zone of high deformation near the fault tip.

Fig. 3.3 Evolution of equivalent strain at $t_3=9E4$ s,CON4

Fig. 3.4 Evolution of equivalent strain at $t_4=11E4$ s,CON4
4. Conclusions

In both CON4 and VER20 cases the pattern of equivalent strain distribution is qualitatively different.

Plots in Fig. 4 of the equivalent strain ($\varepsilon_{eq} = \int_0^\infty \sqrt{\frac{2}{3}} \hat{\varepsilon}_{ij} \hat{\varepsilon}_{ij} \, dt$ with $\hat{\varepsilon}_{ij}$ - strain rate deviator) occurring within some elements along the line between the slope toe and the fault tip show an important influence of the eventual increase of water pressure along a part of the slope surface on the strain magnitude. The increase of pore water pressure may result from heavy rainfall and/or human activity. The fault existence increases the hazard of catastrophic destruction and subsequent landslide due to the occurrence of strain localization.
Acknowledgement This work was supported by the Bulgarian National Science Fund, Grants TN-1304/03 and MM-1301/03. Partly the work on this problem has been done during the visit of the first author in the Department GeoMac of the University of Liege in the frame of the bilateral project (BAS-CGRI) “Coupled THM modeling for multiphase materials and robust methods for model validation and verification”.

References