EFFECT OF HIGHER ORDER ELEMENT ON NUMERICAL INSTABILITY IN TOPOLOGICAL OPTIMIZATION OF LINEAR STATIC LOADING STRUCTURE

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ABSTRACT: This paper presents the mathematical model to solve the topological optimization problem. Effect of higher order element on the optimum topology of the isotropic structure has been studied by using 8-node elements which help in decreasing the numerical instability due to checkerboarding problem in the final topologies obtained. The algorithms are investigated on a number of two-dimensional benchmark problems. MATLAB code has been developed for different numerical two dimensional linear isotropic structure and SIMP approach is applied. Models are discretized using linear quadratic 4-node and 8-node elements and optimal criteria method is used in the numerical scheme. Checkerboarding instability in the final topology is greatly reduces when incorporated 8-node element instead of 4-node element which can be confirmed through comparing the final topologies of the structure.

KEY WORDS: topology optimization; pseudo-densities; optimality criterion; SIMP; checkerboarding.

1 INTRODUCTION

Structural optimization may be referred as the finding the optimum structural design, which is best of all possible design within a prescribed objective and a given set of geometrical or behavioral limitation. However, the best design can be interpreted in different ways, such as having a minimized weight, a maximized stiffness. Structural optimization [1] problem involve finding the state variable such as stresses, displacements etc. and restrictions to the geometry, like a limited amount of material [2, 3]. Hence, there is an interaction between the objective and the constraint. Thus, in structural optimization every property of the structure, like weight, stiffness, critical load, stress, displacement, material properties, geometry or thermal stress [4], can be taken as the objective while taking (parts of) the others as constraints. The typical optimization problem [5–7] in the literature is the so called minimum compliance problem, where the stiffness is maximized (minimizing compliance) with respect to

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a limited amount of material [8]. Structural optimization has become a multidisci-
plinary subject with applications in many fields such as automotive, aeronautical [9],
mechanical, civil, naval engineering [10]. As a result of rapidly growing applications
of structural optimization, the importance of the research in structural optimization
methods is realized by more and more scientists and engineers. Topological structural
optimization is regarded as one of the most challenging topics in structural mechanics,
in which one needs to change the topology as well as the shape during the process
of optimization.

The application of SIMP method in topological optimization problem results in
the problem of non-existence of solutions (ill-posedness) and thus numerical instabil-
ities may occur. One of the most serious numerical instabilities is the occurrence of
checkerboard patterns in the final solutions. This occurs when, around a single node,
there are just two solid elements diagonally connected, as a checkerboard. Another
numerical problem is the fact that different solutions can be obtained just by choos-
ing different number of elements. This is the mesh-dependence problem. In present
investigation effect approach to minimize these numerical instabilities is proposed by
using higher order element. Study involves the application of 8-node element instead
of 4-node to get better optimum topology of the structure by reducing the checker-
boarding problem.

2 Mathematical Formulation of Optimization Problem

The objective of the optimization problem is often some sort of maximization or min-
imization, for example minimization of required time or maximization of stiffness.
To be able to find optimal solution the ‘goodness’ of a solution depending on a par-
ticular set of design variables needs to be expressed with a numerical value. This is
typically done with a function of the design variables. Before conducting a struc-
tural optimization, the qualitative problem description (for example- minimizing the
mass of a cantilever plate in tension subject to maximum stress constraint) must first
be formulated as a quantitative mathematical statement. Any optimization problem
basically consists of the following three types of variables:

Design variable \( (x) \): These are the independent quantities which can be varied to
obtain the optimal design e.g. height and width of a cantilever beam. These are the
variables which control the parameters of component’s design. A particular set of
design variables values represent a particular component design. For example

\[
X^T = [x_1 \ x_2 \ x_3 \ \cdots \ \cdots \ x_{N-1} \ x_N],
\]

where \( x^T \) is a vector of \( N \) design variables.
State variable (y): It refers to the response of a material/structure obtained under given boundary conditions such as stress, displacement force, strain energy, buckling load, moment, etc. These are the equalities and inequalities defining the constraints on the component’s structural behavior. Structural analyses results are required for the evaluation of the constraint or the state variables. Design constraints establish a design’s feasibility. A design which satisfies the constraints is known as a feasible design.

\[
\begin{align*}
\text{Subject to} & \quad \left\{ \begin{array}{c}
g_i(x) \leq 0, \quad i = 1, 2, \ldots, m \\
h_j(x) = 0, \quad j = 1, 2, \ldots, n
\end{array} \right.
\end{align*}
\]

Objective function (f): A function used to classify designs. For every possible design, f returns a number which indicates the goodness of the design. Usually, we choose f such that a small value is better than a large one (a minimization problem). Taking as input, a particular set of design variable values, the objective function converts the values into their corresponding component design, analyzes the design to determine its structural behavior (stresses, deflection, etc.) and then calculates a scalar quantity (volume, mass, etc.) which describes the utility of the design. Hence, the objective function calculates the utility of a set of design variables.

\[
\text{Find } x = \begin{bmatrix}
  x_1 \\
  x_2 \\
  \vdots \\
  x_n
\end{bmatrix}
\text{ which minimizes } f(x).
\]

3 Material and Methods

3.1 SIMP method

In topological optimization, the material of the structure is optimally placed in the given boundary condition and constraint in such a manner to efficiently use the material. It is a mathematical approach to solve the structural problem in such a manner that geometry of structure represents as black-white rendering of an image. In practical problems, we first discretize the structure into finite elements such that each element defines by the certain variable known as pseudo density, denoted by \( \delta_i \) which is the ratio of density of \( i^{th} \) element to the density of solid material. Then optimization of pseudo density is done on the basis of SIMP (solid isotropic material penalization) approach in which nearly “black & white” topology \([6, 11–13]\) is obtained as shown in Fig. 1 on the basis of pseudo density element i.e. value of \( \delta_i \) represented in gray scaling such that elements having \( \delta_i = 1 \) represented as dark gray and \( \delta_i = 0 \) as white. To penalize the intermediate values power law approach is used. Following equations are to
formulate the problem according to SIMP approach:

\[
\begin{align*}
\text{Pseudo density} & \quad \delta_i = \frac{\rho_i}{\rho_0}, \\
\text{Penalization of pseudo density} & \quad \delta_i^p, \\
\text{Formation of stiffness matrix} & \quad K_{\text{new}} = K_i \times \delta_i^p
\end{align*}
\]

(4)

To get optimum topology the final structure should not contain intermediate value of pseudo density for which penalization scheme [14] is been used. Penalization results in reduction of lower value of pseudo density to 0 whereas higher value to 1. This new stiffness matrix is taken to compute the compliance of the structure which results in reduction of lower value of pseudo density to 0 whereas higher value to 1.

For 2-D and 3-D the value of penalization \( p \) is obtained by following equations:

\[
\begin{align*}
(5) \quad p & \geq \max \left\{ \frac{2}{1-\mu^0}, \frac{4}{1+\mu^0} \right\} \quad \text{in 2-D} \\
(6) \quad p & \geq \max \left\{ 15\frac{1-\mu^0}{7-5\mu^0}, \frac{3(1-\mu^0)}{2(1-2\mu^0)} \right\} \quad \text{in 3-D}
\end{align*}
\]

In above equations, the \( \mu^0 \) represent poisson’s ratio and on putting its value 1/3 in 2-D and 3-D equation, the smallest values of \( p \) obtained are 3 and 2, respectively. In present investigation \( p = 3 \) is been used for all cases, one can choose higher value of \( p \).
In present investigation, implementation of a topology optimization based MATLAB code for compliance minimization of statically loaded structures has been performed to determine the effect of higher order on the optimal topologies of the structure and flow chart is shown in Fig. 2. Solution, i.e. optimum value of pseudo density, is obtained in each iteration is presented in the form of a picture. In MATLAB programming the coding for optimal criteria is taken from topological optimization written by Sigmund [15].
3.2 Finite element analysis

In topological optimization problem first design domain is descritized into finite element [16–19] and solution of structural problem is obtained. Such as displacement, stress, natural frequency etc. of each element which act as the state variables for optimization problem. These state variables are indirectly depend upon the design variable i.e. pseudo density. In present investigation MATLAB code for quadrilateral 4-node and 8-node (Table 1 represent shape function) is developed which is used to solve the linear isotropic structural problem of cantilever and MBB beam to determine the displacement of each element which act as the state variable, which is used to determine the compliance of each element and according to which optimization of design variable is done.

Table 1. Shape function of quadrilateral 4-node and 8-node element respectively

<table>
<thead>
<tr>
<th>Quadrilateral 4-node</th>
<th>Quadrilateral 8-node</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ N(\xi, \eta) = \begin{cases} N(\xi, \eta) = 0.25(1-\xi-\eta+\xi\eta) \ N1(\xi, \eta) = 0.25(1-\xi+\xi\eta) \end{cases} ]</td>
<td>[ N(\xi, \eta) = \begin{cases} N1(\xi, \eta) = -0.25 (1-\xi) (1-\eta) (1+\xi+\eta) \ N2(\xi, \eta) = 0.50 (1-\xi^2) (1-\eta) \ N3(\xi, \eta) = -0.25 (1+\xi) (1-\eta) (1-\xi) \ N4(\xi, \eta) = 0.50 (1+\xi) (1-\eta^2) \end{cases} ]</td>
</tr>
<tr>
<td>[ \sigma = \begin{bmatrix} \sigma_x \ \sigma_y \end{bmatrix} ]</td>
<td>[ \sigma = \begin{bmatrix} \sigma_x \ \sigma_y \end{bmatrix} ]</td>
</tr>
</tbody>
</table>

The stiffness matrix for the quadrilateral element having thickness \( t_i \) can be derived from the strain energy in the body, given by

\[
U = \frac{1}{2} \int \sigma^T \varepsilon \, dv, \tag{7}
\]

\[
U = \sum t_i \int \sigma^T \varepsilon \, dA \tag{8}
\]

\[
\{ \sigma \} = [D] \{ B \} \{ a \} \tag{9}
\]

\[
\{ \varepsilon \} = [B] \{ a \}. \tag{10}
\]

where \( \{ a \} \) is the displacement matrix, \( D \) represents stress-strain relationship for 2-D plane stress structure and \( B \) is the strain matrix for 4-node element given as for

\[
[B] = \begin{bmatrix}
\frac{\partial N_1}{\partial \xi} & 0 & \frac{\partial N_2}{\partial \xi} & 0 & \cdots & \frac{\partial N_n}{\partial \xi} & 0 \\
0 & \frac{\partial N_1}{\partial \eta} & 0 & \frac{\partial N_2}{\partial \eta} & \cdots & 0 & \frac{\partial N_n}{\partial \eta} \\
\frac{\partial N_1}{\partial \eta} & \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \cdots & \frac{\partial N_n}{\partial \eta} & \frac{\partial N_n}{\partial \eta}
\end{bmatrix}.
\]
where

\[
U = \sum_{i} \frac{1}{2} t_e \int_{-1}^{1} \int_{-1}^{1} \{a\} \{B\}^T \{D\} \{B\}\{a\} \det J d\xi d\eta,
\]

(12)

\[
= \frac{1}{2} \sum_{i} a^T k a,
\]

(13)

\[
k = t_e \int_{-1}^{1} \int_{-1}^{1} \{B\}^T \{D\} \{B\} \det J d\xi d\eta.
\]

(14)

3.3 Optimality Criteria Method

It is very costly and time consuming to obtain the solution by analytical method because of handling of several variables simultaneously so a new approach known as optimal criteria method is developed. Optimal criteria method [20–24] is numerical approach to solve the topological optimization problem. This approach is based on pseudo-density functions which act the design variable and compliance as objected function. This approach is based on mathematical programming of problem in which state variable are calculated by FE method, which are used to construction problem either by minimizing or maximizing the performance function subjected to different constrains and boundary conditions.

Optimality criteria methods [25] basically divided into two parts, first involve the application of Karush–Kuhn–Tucker conditions which solve the given problem as per required boundary condition and second part consist of optimal criteria approach which is a algorithm to obtain the optimum value of pseudo-density. Both the method as describe below.

3.3.1 Karush–Kuhn–Tucker Conditions

In present investigation objective function is compliance which is to be minimized in a given design domain subjected to certain boundary condition as shown in equation below:

\[
\min_{\delta_i \in \Omega} \text{objective function, } C(\delta_i) = u^T K u = u^T (\sum K_i \times \delta_i^p) u,
\]

(15)

\[
V = \sum_{i=1}^{N} V^T \delta_i \leq V^0,
\]

\[
0 < \delta_{\min} \leq \delta_i \leq 1.
\]

To solve the problem, we first define the Lagrangian function as describe below:
\[ L = u^T K(\delta_i) u + \sum_{i=1}^{m} \lambda_i \left( \sum_{i=1}^{N} V^T \delta_i - V^0 + r_i^2 \right) + \sum_{i=1}^{m} \gamma_i (\delta_{\text{min}} - \delta_i + s_i^2) + \sum_{i=1}^{m} \eta_i (\delta_i - 1 - t_i^2), \]

where \( \lambda_i, \gamma_i \) and \( \eta_i \) are called Lagrange multipliers and \( r, s \) and \( t \) are slack variables.

Solution of above Lagrangian function is obtained by differentiating the function with respect to \( \delta, \lambda, \gamma, \eta, r, s \) and \( t \).

\[ \frac{\partial L}{\partial \delta_i} = \frac{\partial C}{\partial x_i} + \lambda V_i, \]

\[ \frac{\partial L}{\partial \lambda_i} = \sum_{i=1}^{N} V^T \delta_i - V^0 + r_i^2, \]

\[ \frac{\partial L}{\partial \gamma_j} = \delta_{\text{min}} - \delta_i + s_i^2, \]

\[ \frac{\partial L}{\partial \eta_i} = \delta_i - 1 - t_i^2, \]

\[ \frac{\partial L}{\partial r} = 2 \lambda_i r_i, \quad \frac{\partial L}{\partial s} = 2 \gamma_i s_i, \quad \frac{\partial L}{\partial t} = 2 \eta_i t_i. \]

### 3.3.2 Optimal criteria approach

In optimal criteria approach each mesh element is provided with pseudo-density variable which act as design variable in the optimization problem. We have to find the optimum value of pseudo-density which describes the best distribution of material in given design domain which is subjected to certain volume fraction and boundary condition. The element which has lower displacement value is provided lower \( \delta \) values, i.e. no material is required at that position where as for higher displacement elements \( \delta \) value near to one is provided. On the basis of optimal criteria approach mathematical model of the problem can be solved as

\[ B_i = \frac{-dC}{\lambda V_i} = 1. \]

Based on these expressions, the design variables are updated as follows:

\[ \delta_{\text{new}} = \begin{cases} \max(x_{\text{min}}, \delta_i - m) & \text{if } \delta_i B_i^n \leq \max(\delta_{\text{min}}, \delta_i - m), \\ \delta_i B_i^n & \text{if } \max(\delta_{\text{min}}, \delta_i - m) < \delta_i B_i^n < \min(1, \delta_i + m) \\ \min(1, \delta_i + m) & \text{if } \min(1, \delta_i + m) \leq \delta_i B_i^n \end{cases} \]
where, $m$ represents the move limit which is the allowable change in $\delta_i$ and generally taken as 0.2. Bisection method is used to determination the value of Lagrange constant $\lambda$.

### 3.4 Numerical difficulties

The density-based formulation with penalization leads to several numerical difficulties. Over the years, different mathematical model had evolved to eliminate or suppress these phenomena. This section involves the two main numerical challenges which act as the limitation for SIMP-related methods and presents a brief study including some discussion and evaluation of the various approaches taken to address each problem. In present investigation use of higher order element to overcome these changes has been presented. The results obtained using 2-D 4node element was compared with 8-node element, with a uniform meshing. The optimization was performed using the SIMP power law combined with the optimality criteria method for compliance minimization subject to a volume constraint.

One of the major problems in topological optimization is Checkerboarding pattern [26, 27] as shown in Fig. 3 in which alternative solid and void elements region occurs, which is undesirable to engineer. These occur due to poor numerical modeling of the stiffness of chequerboard patterns. Chequerboards occur most commonly in models that use 4-nodes linear quadrilateral elements, where all forces acting on an element can be transferred completely through point-connections at the corner nodes. Therefore, when using these elements, the chequerboard pattern exhibits an artificially high stiffness. Various methods have been proposed to lessen Checkerboarding. In present investigation implementations of higher order, i.e., 8-node over 4-node elements is preferred.

Mesh-dependency refers to the property through which the level of coarseness or fineness of the finite element mesh affects the number of members and the overall complexity of the optimized structure. Unless steps are taken to reduce this effect, a fine mesh will yield a large number of excessively thin members, which reduces the viability and manufacturability of the optimized structure. This occurs because the generalized physical description of the problem is ill-posed and therefore suffers from the nonexistence of solutions. Fig. 4 represent different topologies obtained at different mesh density in ANSYS software. In present investigation effect of higher order element on the topologies of same structure at different mesh density has been studied.

![Fig. 3. Chequerboard pattern appearing in the solution of a cantilevered beam [28].](image-url)
Mesh-dependency refers to the property through which the level of coarseness or fineness of the finite element mesh affects the number of members and the overall complexity of the optimized structure. Unless steps are taken to reduce this effect, a fine mesh will yield a large number of excessively thin members, which reduces the viability and manufacturability of the optimized structure. This occurs because the generalized physical description of the problem is ill-posed and therefore suffers from the nonexistence of solutions. Figure 4 represent different topologies obtained at different mesh density in ANSYS software. In present investigation effect of higher order element on the topologies of same structure at different mesh density has been studied.

4 RESULT AND DISCUSSIONS
4.1 SPECIMEN GEOMETRY USED

In the present investigation two specimens have been used. Geometry and boundary conditions of both the specimens are as shown in Fig. 5. The specimens are linear isotropic structure subjected to the state of plain stress having thickness of unity for
static loading condition. Intel core i5, 2.5 GHz processor with 4 GB DDR3 RAM is used to perform the numerical analysis and MATLAB 64-bit R2010b is used to developed and execute the code. Table 2 presents the input parameter for both numerical problems. Model 1 is a cantilever beam fixed at left edge and point load of 1 N is applied at lower right corner whereas Model 2 is half Messerschmitt Bolkow Blohm beam which fixed from y-direction at lower right corner and form x-direction at the left edge. A point load of 1 N is applied at the upper left corner. Value of load and Young’s modulus are taken 1 to reduce the computation time and doesn’t affect the final topology.

Table 2. Input parameter for numerical problems

<table>
<thead>
<tr>
<th></th>
<th>Young’s modulus (E)</th>
<th>Poisson’s ratio (ν)</th>
<th>Load (N)</th>
<th>Volume fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>MODEL-1</td>
<td>1 N/m</td>
<td>0.3</td>
<td>1</td>
<td>0.5</td>
</tr>
<tr>
<td>MODEL-2</td>
<td>1 N/m</td>
<td>0.3</td>
<td>1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

4.2 EFFECT OF HIGHER ORDER ELEMENT ON CHECKERBOARDING AND MESH DENSITY

In present investigation an alternative approach to overcome the numerical difficulties such as checkerboarding and mesh density has been presented by incorporating higher order element in topological optimization problem. To improve the final optimal topology of structure, the effect of higher order element, i.e. is 8-node element over 4-node element are studied with the aim of reduction in possible difficulties in getting pure 0-1 optimal design arises due to checkerboarding problem. In this section comparison between optimum shapes obtained by topological optimization using quadrilateral 4-nodes and quadrilateral 8-nodes through MATLAB based optimal criteria method is done. To study the effect two models have been study: Model 1 (cantilever beam) and Model 2 (Messerschmitt Bolkow Blohm beam) as shown in Fig. 5.

Comparison between optimal topologies obtained at different mesh densities are presented in Fig. 6. Left side of the figure represents final topologies obtained with 4-node quadrilateral element whereas right side represent with 8-node quadrilateral element. Result shows that structures mesh with 4-nodes element have higher checkerboarding problem in final topology as compared with 8-node element. In 8-node element case final topologies have almost same design which shows that checkerboarding problem is highly reduced and the effect of mesh density is very less. From these results we can conclude that checkerboarding problem can be solved by using higher order element.
Fig. 6. Comparing the optimal shapes obtained by MATLAB OC between 4-node and 8-node element for Cantilever beam (b) Messerschmitt Bolkow Blohm beam.

Figure 7 represents graphically the variation of compliance and volume fraction. Table 3 represents the detail result obtained. At mesh density of $26 \times 15$ final compliance value obtained is 45.12 Nmm after 29 iterations and time taken is 5 s for 4-node

<table>
<thead>
<tr>
<th>Mesh density</th>
<th>4-node element Compliance (Nmm)</th>
<th>Iterations</th>
<th>8-node element Compliance (Nmm)</th>
<th>Iterations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$24 \times 15$</td>
<td>45.12</td>
<td>29</td>
<td>49.92</td>
<td>28</td>
</tr>
<tr>
<td>$32 \times 20$</td>
<td>45.15</td>
<td>29</td>
<td>49.84</td>
<td>36</td>
</tr>
<tr>
<td>$40 \times 25$</td>
<td>46.00</td>
<td>32</td>
<td>49.85</td>
<td>81</td>
</tr>
<tr>
<td>Model 2</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$36 \times 20$</td>
<td>62.92</td>
<td>60</td>
<td>73.44</td>
<td>40</td>
</tr>
<tr>
<td>$48 \times 20$</td>
<td>117.76</td>
<td>33</td>
<td>131.11</td>
<td>40</td>
</tr>
<tr>
<td>$60 \times 20$</td>
<td>203.24</td>
<td>65</td>
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<td>43</td>
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</table>
4.2. EFFECT OF HIGHER ORDER ELEMENT ON CHECKERBOARDING AND MESH DENSITY

In the present investigation, an alternative approach to overcome numerical difficulties such as checkerboarding and mesh density has been presented by incorporating higher order elements into the topological optimization problem. To improve the final optimal topology of the structure, the effect of higher order elements, specifically the 8-node element over the 4-node element, was studied with the aim of reducing the possible difficulties in obtaining pure 0-1 optimal design due to the checkerboarding problem. In this section, the comparison between optimum shapes obtained by topological optimization using quadrilateral 4-node and 8-node elements through MATLAB-based optimal criteria methods is done.

For the 4-node element, the compliance value is 49.92 Nmm after 28 iterations and the time taken is 23 s. Computational time highly increases at higher mesh density for the 8-node element, i.e., time taken for $32 \times 20$ and $40 \times 20$ mesh density for the 4-node element is 13 s and 38 s, whereas for the 8-node element is 118 s and 990 s respectively. And also with an increase in mesh density, the number of iterations for the 8-node element is higher as compared to the 4-node element. These may act as drawbacks for using higher order elements, but on comparing the optimal topologies, there is a drastic reduction in the checkerboarding due to which these drawbacks can be neglected.
The final optimal topologies of half Messerschmitt Bolkow Blohm beam with 4-nodes and 8-nodes quadrilateral are represented in Fig. 8. As the final topology obtained by using 4-nodes and 8-nodes element show that the topology obtained by using 4-nodes element contain large amount of Checkerboarding problem as compare to 8-nodes element as these Checkerboards area is non-desirable from above observation it can be conclude that using of higher nodes element in topological optimization decrease numerical instability in the final optimal shape. The final optimal compliance for half Messerschmitt Bolkow Blohm beam with 4-node and 8-node quadrilateral is presented in Table 3. As per the result obtained compliance value for 8-node is higher than the 4-node element as shown in Fig. 9 graphically, the following observation shows that as the number of nodes increases the stiffness of structure is decreased as compliance is inversely proportional to stiffness it value is increased. The shape function of 8-node element contain cubic interpolation as shown in Table 2 which provide better accuracy in solution, further the extra nodes on each edge center provide better connectivity between element due to which numerical instabilities such as checkerboarding and mesh dependency are highly reduces.
Table 3: Comparison between optimal compliance and iterations

<table>
<thead>
<tr>
<th>Mesh density</th>
<th>4-node element</th>
<th>8-node element</th>
</tr>
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</tr>
<tr>
<td>46.00</td>
<td>32</td>
<td>49.85</td>
</tr>
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</table>

Fig. 9. Graph plot for compliance value for half Messerschmitt Bolkow Blohm beam.

5 Conclusion

Present investigation shows the application of SIMP method and optima criteria for solving topological optimization problem of linear elastic isotropic structure. Effect of higher order element of numerical stability like checkerboarding problem has been studied with the benchmark structures. An algorithm has been presented to overcome numerical instability in topological optimization problem in terms of checkerboarding through MATLAB code. Optimum topologies of structure obtained using 8-node element are compared with topologies obtained with 4-node and result shows that
application of higher order element results in reduction of checkerboarding problem to great extend.

REFERENCES


