ABSTRACT: This paper presents an analysis for a micro/nano wedge-platform thrust slider bearing by using the flow factor approach model. The contact-fluid interfacial shear strength was taken into account for describing the interfacial slippage. The carried load and friction coefficient of the bearing were calculated when different contact-fluid interactions were used. It was found that the interaction strength between the contact and the fluid has a significant contribution to the load-carrying capacity of the bearing, a weak contact-fluid interaction in the bearing inlet zone and its resulting interfacial slippage on the stationary contact surface is beneficial for both the load-carrying capacity and the friction coefficient of the bearing, while a strong contact-fluid interaction in the bearing outlet zone is contrarily harmful. The relative slip amount is linearly distributed in the bearing inlet zone, when the interfacial slippage occurs on the stationary surface in this subzone because of the low interfacial shear strength.

KEY WORDS: bearing; load; friction; interfacial slippage; interfacial shear strength.

1 INTRODUCTION
Micro/nano bearings are important in supporting the load and reducing the friction and wear in micro mechanical systems [1]. Conventional lubrication analysis fails for a micro/nano bearing, because the lubricating media in this bearing is a physical adsorbed layer and conventional lubrication theory did not consider this layer [2]. Both the Couette flow and the Poiseuille flow in a nano channel have been found to deviate from continuum flow theory [3–6]. The inhomogeneous density distribution across the channel height is largely responsible for such deviations. It has also been found that the fluid may often slip on a wall in a nano channel when the fluid-wall
interaction is weak [7–9]. A suitable lubrication theory for a micro/nano bearing should account for all these mentioned factors.

Zhang and his colleague proposed the flow factor approach model for describing the flow in a nano gap [10, 11]. Their model was well validated because of good matching with molecular dynamics simulation results for both the Couette and Poiseuille flows in a nano channel [12–15]. This model incorporates the interfacial slippage, non-continuum and dynamic effects of the confined fluid. Zhang used this model to analyze the performance of a nano step bearing [16, 17]. Recently, he presented an analysis for a nano bearing formed between two sliding parallel plane surfaces owing to physical adsorption [18]. His obtained results are physically reasonable, although further experiments may need to verify them.

Li et al. calculated the carried load and friction coefficient of a hydrodynamic lubricated wedge-platform thrust slider bearing (in macro size) when the artificial interfacial slippage was introduced, by using continuum lubrication theory [19]. Qian et al. analyzed a micro/nano wedge-platform thrust slider bearing with small tilting angles respectively considering homogeneous and inhomogeneous wall surfaces, by using the flow factor approach model [20, 21]. Their analysis used the parameter of the relative slip amount to account for the fluid-wall interfacial slippage. It is obvious that the magnitude of the relative slip amount depends on the fluid-wall interfacial shear strength. More direct results would be obtained in their analysis if the fluid-wall interfacial shear strength had been introduced to account for the interfacial slippage.

In this paper, an analysis was presented for a micro/nano wedge-platform thrust slider bearing based on the fluid-wall interfacial shear strength, by using the flow factor approach model. Different interactions between the fluid and the wall respectively in the bearing inlet and outlet zones were considered. The carried load and friction coefficient of the bearing were calculated for different operating conditions. The relative slip amount was calculated for the zone where the interfacial slippage occurred. Its distributions were clearly shown for different fluid-wall interfacial shear strengths.

2 Studied Bearing

The studied micro/nano bearing is shown in Fig. 1. In this bearing, the upper contact surface is stationary, while the lower contact surface is moving with the speed \( u \). The lubricated area of the bearing is divided into the “I” and ‘II’ sub zones, namely the outlet and inlet zones respectively. The boundary slippage is artificially designed on the stationary contact surface in the inlet zone, where the contact-fluid interfacial shear strength \( \tau_{sa} \) is low. On the other bearing surfaces boundary slippage is absent. \( h_i \) and \( h_o \) are respectively the lubricating film thicknesses on the entrance and exit of the bearing. \( l_1 \) and \( l_2 \) are respectively the widths of the inlet and outlet zones of the bearing. The coordinates used are also shown in Fig. 1.
In the two sub zones, the stationary contact surface may respectively have different interactions with the lubricating fluid. Table 1 gives the symbols used in the paper marking different interactions between the fluid and the stationary contact surface respectively in the two sub zones.

Table 1. The symbols used in the paper making different interactions between the fluid and the stationary contact surface respectively in the ‘I’ and ‘II’ sub zones

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Sub zone</th>
</tr>
</thead>
<tbody>
<tr>
<td>S-M</td>
<td>Strong</td>
</tr>
<tr>
<td>S-W</td>
<td>Strong</td>
</tr>
<tr>
<td>M-W</td>
<td>Medium</td>
</tr>
</tbody>
</table>

3 Analysis
3.1 Analysis for the ‘I’ Sub Zone

The Reynolds equation for the ‘I’ sub zone is [14, 16]

\[
q_{m, bf} = \frac{u}{2} \frac{h o \rho_{bf,I}(h_o)}{12} \frac{d p(x)}{dx} + \frac{S_I(h_o) \eta_{bf,I}(h_o) h_o^3}{\nu_{bf,I}(h_o)} ,
\]

where \( p \) is the lubricating film pressure, \( q_{m, bf} \) is the mass flow rate per unit contact length through the bearing, \( \rho_{bf,I} \) and \( \eta_{bf,I} \) are respectively the average density across the film thickness and effective viscosity of the confined fluid in the ‘I’ sub zone, and \( S_I \) is the parameter describing the non-continuum effect of the confined fluid in the ‘I’ sub zone.
By using the boundary condition \( p|_{x=0} = 0 \), integrating equation (1) gives the film pressure in the ‘I’ sub zone

\[
(2) \quad p = \frac{12 \eta_{eff}^{bf,I}(q_{m,bf} + u_h \rho_{eff}^{bf,I}/2)}{S_I \rho_{eff}^{bf,I} h_o^3} x \quad \text{for} \quad 0 \leq x \leq l_2
\]

The film pressure at \( x = l_2 \) is

\[
(3) \quad p = \frac{12 \eta_{eff}^{bf,I}(q_{m,bf} + u_h \rho_{eff}^{bf,I}/2)}{S_I \rho_{eff}^{bf,I} h_o^3} l_2.
\]

### 3.2 Analysis for the ‘II’ Sub Zone

#### 3.2.1 Basic Equations

The Reynolds equation for the ‘II’ sub zone is:

\[
q_{m,bf} = \bar{\nu}_{II} h(x) \rho_{eff}^{bf,II}(h_{m,II}) + \frac{S_{II}(h_{m,II}) \eta_{eff}^{bf,II}(h_{m,II}) h^3(x)}{12 \eta_{eff}^{bf,II}(h_{m,II})} \frac{dp(x)}{dx},
\]

where \( \rho_{eff}^{bf,II} \) and \( \eta_{eff}^{bf,II} \) are respectively the average density across the film thickness and the effective viscosity of the confined fluid in the ‘II’ sub zone, \( S_{II} \) is the parameter describing the non-continuum effect of the confined fluid in the ‘II’ sub zone, the mean film thickness \( h_{m,II} \) in the “II” sub zone is \( h_{m,II} = h_o + l_1 \tan \theta/2 \), and \( \bar{\nu}_{II} = (\bar{u}_a - u)/2 \). Here, \( \bar{u}_a \) is the speed of the lubricating film at the stationary surface in the inlet zone.

The shear stress of the equivalent continuum film in the present study is [10]

\[
(5) \quad \tau = \eta_{eff}^{bf,II} \frac{\partial v}{\partial z},
\]

where \( v \) is the film velocity. According to Eq. (5) and the momentum equilibrium equation of this equivalent continuum film, the velocity distribution function across the film thickness of the equivalent continuum film is derived as [17]

\[
(6) \quad v = \frac{1}{2 \eta_{eff}^{bf,II} x} \frac{dp}{dx} z^2 + v_1 z + v_2,
\]

where \( v_1 \) and \( v_2 \) are constants. From the boundary condition \( v|_{z=0} = -u \), it is solved that \( v_2 = -u \).

The shear stress of the equivalent continuum film at the upper contact surface in the ‘II’ sub zone is [17]

\[
(7) \quad \tau'_{a,II} = \frac{dp}{dx} h + v_1 \eta_{eff}^{bf,II},
\]
where $h$ is the film thickness in the ‘II’ sub zone. According to Zhang’s earlier research [10], this shear stress should be corrected by the factor $\theta_{\tau,a,II}$ to give the real shear stress of the film at the upper contact surface because of the film non-continuum effect. The shear stress of the film at the upper contact surface in the ‘II’ sub zone is then

$$\tau_{a,II} = \theta_{\tau,a,II} \tau'_a = \tau_{sa}. \quad (8)$$

The factor $\theta_{\tau,a,II}$ is dependent on the film thickness and here taken as [17]

$$\theta_{\tau,a,II}(H_{m,II}) = k_0 (H_{m,II} - 1)^2 + 1 \quad \text{for} \quad 0 < H_{m,II} < 1, \quad (9)$$

where $H_{m,II} = h_{m,II}/h_{cr,bf,II}$, and $k_0$ is constant.

Substituting Eq. (7) into Eq. (8) and rearranging gives

$$c_1 = \frac{\tau_{sa}}{\theta_{\tau,a,II} \eta_{bf,II}^\text{eff}} - \frac{h}{\eta_{bf,II}^\text{eff}} \frac{dp}{dx}. \quad (10)$$

In the ‘II’ sub zone, since $\bar{u}_a = \frac{\tau_{sa} h}{2 \eta_{bf,II}^\text{eff} \theta_{\tau,a,II}} - \frac{h^2}{2 \eta_{bf,II}^\text{eff}} \frac{dp}{dx} - u$, $\bar{u}_{II}$ is [17]

$$\bar{u}_{II} = \frac{1}{2 \eta_{bf,II}^\text{eff} \theta_{\tau,a,II}} h - \frac{1}{4 \eta_{bf,II}^\text{eff} \theta_{\tau,a,II}} \frac{dp}{dx} h^2 - u. \quad (10)$$

Substituting Eq. (10) into Eq. (4) gives the following Reynolds equation for the ‘II’ sub zone [17]:

$$\frac{dp}{dx} = \frac{\eta_{bf,II}^\text{eff} (\eta_{m,bf} + u h \rho_{bf,II}^\text{eff} - \tau_{sa} \rho_{bf,II}^\text{eff} h^2 / 2 \rho_{bf,II}^\text{eff} \theta_{\tau,a,II})}{\rho_{bf,II}^\text{eff} \lambda_s h^3}, \quad (11)$$

where $\lambda_s = S_{II}/12 - 1/4$, the film thickness $h$ is expressed as

$$h = h_o + \frac{h_i - h_o}{l_1} (x - l_2). \quad (12)$$

By using the boundary condition $p|_{x=l_1+l_2} = 0$, integrating equation (11) gives the film pressure in the “II” sub zone

$$p = \frac{1}{k} \left[ \frac{\eta_{m,bf} \eta_{bf,II}^\text{eff}}{\rho_{bf,II}^\text{eff} \lambda_s} \left( \frac{1}{2 h_i^2} - \frac{1}{2 h^2} \right) + \frac{u h \rho_{bf,II}^\text{eff}}{\lambda_s} \left( \frac{1}{h_i} - \frac{1}{h} \right) + \frac{\tau_{sa}}{2 \theta_{\tau,a,II} \lambda_s} \ln \frac{h_i}{h} \right]$$

for $l_2 \leq x \leq l_1 + l_2$, \quad (13)
where \( k = \tan(\theta) \).

According to equation (13), the film pressure at \( x = l_2 \) is

\[
P = \frac{1}{k} \left[ \frac{q_{m,bf} \eta_{bf,II}}{\rho_{bf,II} \lambda_s} \left( \frac{1}{2 h_i^2} - \frac{1}{2 h_o^2} \right) + \frac{\eta_{bf,II}}{\lambda_s} \left( \frac{1}{h_i} - \frac{1}{h_o} \right) + \frac{\tau_{sa}}{2 \theta_{\tau,a,II} \lambda_s} \ln \frac{h_i}{h_o} \right].
\]

Solving the coupled equations (3) and (14) gives

\[
q_{m,bf} = \frac{6 \eta_{bf,II}^\text{eff} l_2}{S_I h_o^3} - \frac{1}{k} \left[ \frac{\eta_{bf,II}}{\lambda_s} \left( \frac{1}{h_i} - \frac{1}{h_o} \right) + \frac{\tau_{sa}}{2 \theta_{\tau,a,II} \lambda_s} \ln \frac{h_i}{h_o} \right].
\]

### 3.2.2 Relative Slip Amount

Zhang introduced the relative slip amount to describe the interfacial slippage in the flow factor approach model [18]. He formulated

\[
\bar{\gamma}_{s,II} = -\gamma_{s,II} - \frac{1}{u} = \gamma_{s,II} + \frac{1}{\eta_{bf,II}^\text{eff} (h_m,II) \eta_{bf,II}^\text{eff} (h_m,II) u} \left( \frac{1}{4 \lambda_s} + 1 \right) + 1.
\]

### 3.3 Carried Load

The carried load per unit contact length of the bearing is

\[
w_{bf, \text{slip}} = \int_0^{l_2} p \, dx + \int_{l_2}^{l_1 + l_2} p \, dx = \left( \frac{6 q_{m,bf} \eta_{bf,II}}{S_I \rho_{bf,II} h_o^3} + \frac{3 \eta_{bf,II}^\text{eff}}{S_I h_o^3} \right) l_2^2
\]

\[
+ \frac{1}{k^2} \left[ q_{m,bf} \eta_{bf,II}^\text{eff} \left( \frac{2}{h_i} - \frac{h_o}{h_i^2} - \frac{1}{h_o} \right) + \frac{\eta_{bf,II}^\text{eff}}{\lambda_s} \left( 1 - \frac{h_o}{h_i} + \ln \frac{h_o}{h_i} \right)
\]

\[
+ \frac{\tau_{sa}}{2 \theta_{\tau,a,II} \lambda_s} \left( h_i - h_o - h_o \ln \frac{h_i}{h_o} \right) \right].
\]

### 3.4 Friction Coefficient and Interfacial Slipping Velocity

The shear stress of the equivalent continuum film at the upper contact surface in the ‘I’ sub zone is

\[
\tau_{a,I} = \frac{\eta_{bf,II}^\text{eff}}{h_o} + \frac{6 q_{m,bf} \eta_{bf,II}^\text{eff}}{S_I \rho_{bf,II} h_o^3} + \frac{3 \eta_{bf,II}^\text{eff}}{S_I h_o^3}.
\]
The real shear stress at the upper contact surface in the ‘I’ sub zone is [10]:

\[
\tau_{a,I} = \theta_{\tau,a,I}\tau_{a,I}^\prime,
\]

where \(\theta_{\tau,a,I}\) is the correction factor accounting for the film non-continuum effect, and it is here taken as [17]

\[
\theta_{\tau,a,I}(H_o) = k_0(H_o - 1)^2 + 1 \quad \text{for} \quad 0 < H_o < 1,
\]

where \(H_o = h_o/h_{cr,bf,I}\).

For preventing the boundary slippage occurrence on the upper contact surface in the ‘I’ sub zone, it is required that \(\tau_{a,I} < \tau_{sa,I}\). Here, \(\tau_{sa,I}\) is the contact-fluid interfacial shear strength at the upper contact surface in the ‘I’ sub zone.

The shear stress of the equivalent continuum film at the lower contact surface in the ‘I’ sub zone is

\[
\tau_{b,I}^\prime = \frac{u \eta_{bf,I}}{h_o} - \frac{6q_{m,bf}\eta_{bf,I}}{S_I\rho_{bf,I}h_o^2} - \frac{3u \eta_{bf,I}}{S_I h_o}.
\]

The real shear stress at the lower contact surface in the ‘I’ sub zone is then [10]

\[
\tau_{b,I} = \theta_{\tau,b,I}\tau_{b,I}^\prime,
\]

where the correction factor \(\theta_{\tau,b,I}\) is taken as \(\theta_{\tau,b,I} = \theta_{\tau,a,I}\).

For preventing the boundary slippage occurrence on the lower contact surface in the ‘I’ sub zone, it should be satisfied that \(\tau_{b,I} < \tau_{sb}\). Here, \(\tau_{sb}\) is the contact-fluid interfacial shear strength on the whole lower contact surface.

The real shear stress at the lower contact surface in the ‘II’ sub zone is [10]

\[
\tau_{b,II} = \theta_{\tau,b,II}\tau_{b,II}^\prime = \theta_{\tau,b,II}\left[\frac{\tau_{sa}}{\theta_{\tau,a,II}} \left(1 + \frac{1}{2\lambda_s}\right) - \frac{q_{m,bf}\eta_{bf,II}}{\rho_{bf,II}^\prime h_s^2} - \frac{u \eta_{bf,II}}{\lambda_s h}\right],
\]

where the correction factor \(\theta_{\tau,b,II}\) is taken as \(\theta_{\tau,b,II} = \theta_{\tau,a,II}\).

The friction force per unit contact length at the upper contact surface in the bearing is

\[
F_{f,a} = \int_0^{l_2} \tau_{a,I}dx + \int_{l_2}^{l_1+l_2} \tau_{sa}dx
= \theta_{\tau,a,I}\left(\frac{u \eta_{bf,I}}{h_o} + \frac{6q_{m,bf}\eta_{bf,I}}{S_I\rho_{bf,I}h_o^2} + \frac{3u \eta_{bf,I}}{S_I h_o}\right)l_2 + \tau_{sa}l_1.
\]
The friction force per unit contact length at the lower contact surface in the bearing is

\[
F_{f,b} = \int_0^{l_2} \tau_{b,I} dx + \int_{l_2}^{l_1+l_2} \tau_{b,II} dx
= \theta_{\tau,b,I} \left( \frac{u_{\eta,\text{eff},b,I}}{h_o} - \frac{6q_{m,bf}\eta_{\text{eff},b,I}}{S_I\rho_{b,f,II}h_o^2} - \frac{3u_{\eta,\text{eff},b,I}}{S_Ih_o} \right) l_2
+ \frac{\theta_{\tau,b,II}}{k} \left[ \frac{\tau_{sa}}{\theta_{\tau,a,II}} (h_1 - h_o) \left( 1 + \frac{1}{2\lambda_s} \right) 
+ \frac{q_{m,bf}\eta_{\text{eff},b,II}}{\rho_{b,f,II}h_s} \left( \frac{1}{h_i} - \frac{1}{h_o} \right) - \frac{u_{\eta,\text{eff},b,II}}{\lambda_s} \ln \frac{h_i}{h_o} \right].
\]

The friction coefficients at the upper and lower contact surfaces in the bearing are respectively:

\[
f_{a,bf,\text{slip}} = \frac{F_{f,a}}{w_{bf,\text{slip}}}, \quad \text{and} \quad f_{b,bf,\text{slip}} = \frac{F_{f,b}}{w_{bf,\text{slip}}}.\]

The film slipping velocity on the upper contact surface in the ‘II’ sub zone, which is along the \(x\) coordinate is finally expressed as

\[
\Delta u_{a,x} = \frac{\tau_{sa} h}{\theta_{\tau,a,II}\rho_{b,f,II}} \left( 1 + \frac{1}{4\lambda_s} \right) - \frac{q_{m,bf}\eta_{\text{eff},b,II}}{2\rho_{b,f,II}h_s} - u \left( 1 + \frac{1}{2\lambda_s} \right).
\]

For ensuring the occurrence of the film slippage on the upper contact surface but preventing the film slippage occurrence on the lower contact surface in the ‘II’ sub zone, it should be satisfied that

\[
\Delta u_{a,x} < 0, \quad |\tau_{b,II}| < \tau_{sb}.
\]

4 Normalization

4.1 For the present bearing

The obtained results are here normalized and the following dimensionless parameters are defined

\[
\psi = \frac{l_1}{l_2}, \quad \alpha = \frac{h_o}{l_1+l_2}, \quad X = \frac{x}{l_1+l_2}, \quad \lambda_h = \frac{h_i}{h_o}, \quad \lambda = \frac{h}{h_o}, \quad H_o = \frac{h_o}{h_{cr,bf,I}},
\]

\[
H_{m,II} = \frac{h_{m,II}}{h_{cr,bf,II}}, \quad Cq_I(H_o) = \frac{\rho_{b,f,I}(h_o)}{\rho_a}, \quad Cq_{II}(H_{m,II}) = \frac{\rho_{b,f,II}(h_{m,II})}{\rho_a}.
\]
\[
C_y I(H_o) = \frac{\eta_{bf, I}(h_o)}{\eta_a}, \quad C_y II(H_{m, II}) = \frac{\eta_{bf, II}(h_{m, II})}{\eta_a}, \quad P = \frac{ph_o}{u\eta_a},
\]
\[
W = \frac{w}{u\eta_a}, \quad Q_{m, bf} = \frac{q_{m, bf}}{uh_o\eta_a}, \quad \bar{\tau}_{sa} = \frac{\tau_{sa}h_o}{u\eta_a}, \quad \bar{\tau} = \frac{\tau h_o}{u\eta_a}, \quad \bar{F}_{f, a} = \frac{F_{f, a}}{u\eta_a}, \quad \bar{F}_{f, b} = \frac{F_{f, b}}{u\eta_a}, \quad DU = \frac{\Delta u_{a, x}}{u}.
\]

The dimensionless mass flow rate per unit contact length through the bearing is

\[
(29) \quad Q_{m, bf} = \left( \frac{6C_y I(H_o)Q_{m, bf}}{S_I(H_o)\alpha(1 + \psi)} - \frac{1}{k} \left[ \frac{C_y II(H_{m, II})}{\lambda_s} \left( \frac{1}{\lambda_h} - 1 \right) + \frac{\bar{\tau}_{sa}}{2\lambda_s\theta_{\tau, a, II}(H_{m, II})\ln \lambda_h} \right] \right) \times \left( \frac{1}{k} \left[ \frac{C_y II(H_{m, II})^2}{2C_q II(H_{m, II})\lambda_s} \left( \frac{1}{\lambda_h^2} - 1 \right) - \frac{1}{S_I C_q I(H_o)\alpha} \right] \right)^{-1}.
\]

The dimensionless pressure in the ‘I’ sub zone is

\[
(30) \quad P = \left( \frac{12C_y I(H_o)Q_{m, bf}}{S_I(H_o)C_y I(H_o)} \right) X \alpha \quad \text{for} \quad 0 < X < \frac{1}{\psi + 1}.
\]

The dimensionless pressure in the ‘II’ sub zone is

\[
(31) \quad P = \frac{1}{k} \left[ \frac{Q_{m, bf}C_y II(H_{m, II})}{2C_q II(H_{m, II})\lambda_s} \left( \frac{1}{\lambda_h^2} - \frac{1}{\lambda^2} \right) + \frac{C_y II(H_{m, II})}{\lambda_s} \left( \frac{1}{\lambda_h} - \frac{1}{\lambda} \right) + \frac{\bar{\tau}_{sa}}{2\lambda_s\theta_{\tau, a, II}(H_{m, II})\ln \lambda_h} \right] \quad \text{for} \quad \frac{1}{\psi + 1} < X < 1.
\]

The dimensionless load carried by the bearing is

\[
(32) \quad W_{bf, slip} = \left( \frac{6Q_{m, bf}C_y I(H_o)}{S_I(H_o)C_q I(H_o)} + 3C_y I(H_o) \right) \left( \frac{1}{\alpha(1 + \psi)} \right)^2 \quad + \frac{1}{k^2} \left( \frac{Q_{m, bf}C_y II(H_{m, II})}{2C_q II(H_{m, II})\lambda_s} \left( \frac{2}{\lambda_h} - \frac{1}{\lambda_h^2} - 1 \right) + \frac{C_y II(H_{m, II})}{\lambda_s} \left( 1 - \frac{1}{\lambda_h} + \ln \frac{1}{\lambda_h} \right) \right) \right) \left( \frac{\bar{\tau}_{sa}}{2\lambda_s\theta_{\tau, a, II}(H_{m, II})\lambda_h - 1 - \ln \lambda_h} \right).
\]
The relative slip amount in the ‘II’ sub zone is

\[ \gamma_{s,II} = \frac{Q_{m,bf}}{2C_{yII}(H_{m,II})\lambda_s} + \frac{1}{2\lambda_s} - \bar{\tau}_{sa}\lambda \theta_{r,a,II}(H_{m,II})C_{yII}(H_{m,II}) \left( \frac{1}{4\lambda_s} + 1 \right) + 1. \]

The dimensionless friction force per unit contact length on the upper contact surface in the bearing is

\[ \bar{F}_{f,a} = \frac{\theta_{r,a,I}(H_o)C_{yI}(H_o)}{\alpha(1+\psi)} \left( 1 + \frac{6Q_{m,bf}}{S_I(H_o)C_{yI}(H_o)} + \frac{3}{S_I(H_o)} \right) + \frac{\bar{\tau}_{sa}}{\alpha(1+\frac{1}{\psi})}. \]

The dimensionless friction force per unit contact length on the lower contact surface in the bearing is

\[ \bar{F}_{f,b} = \frac{\theta_{r,a,I}(H_o)C_{yI}(H_o)}{\alpha(1+\psi)} \left( 1 - \frac{6Q_{m,bf}}{S_I(H_o)C_{yI}(H_o)} - \frac{3}{S_I(H_o)} \right) + \frac{\bar{\tau}_{sa}k}{\theta_{r,a,II}(H_{m,II})} \left( \frac{1}{2\lambda_s} \right) \left( \lambda_h - 1 \right) + Q_{m,bf}C_{yII}(H_{m,II}) \left( \frac{1}{\lambda_s} - 1 \right) - \frac{C_{yII}(H_{m,II})}{\lambda_s} \ln \lambda_h. \]

The friction coefficients on the upper and lower contact surfaces in the bearing are respectively

\[ f_{a,bf,slip} = \frac{\bar{F}_{f,a}}{W_{bf,slip}} \quad \text{and} \quad f_{b,bf,slip} = \frac{\bar{F}_{f,b}}{W_{bf,slip}}. \]

The dimensionless film slipping velocity on the upper contact surface in the ‘II’ sub zone is

\[ DU = \frac{\bar{\tau}_{sa}\lambda}{C_{yII}(H_{m,II})\theta_{r,a,II}(H_{m,II})} \left( 1 + \frac{1}{4\lambda_s} \right) - \frac{Q_{m,bf}}{2C_{yII}(H_{m,II})\lambda_s} \frac{1}{2\lambda_s} - 1. \]

4.2 For conventional results

When no boundary slippage is considered, according to conventional (continuum) hydrodynamic lubrication theory [2], the dimensionless carried load per unit contact length of the bearing in Fig. 1 is
Performance of a Micro/Nano Wedge-Platform Thrust Slider Bearing Based on...

\[
W_{\text{conv}} = \frac{6}{\alpha^2} \left\{ \frac{\psi}{\psi + 1} \right\}^2 \left[ \frac{\ln \lambda_h}{(\lambda_h - 1)^2} - \frac{2}{\lambda_h^2 - 1} \right] + \frac{\psi(\lambda_h - 1)}{(\psi + 1)^2(2\lambda_h^2 + \psi(\lambda_h + 1))} \left[ \frac{\psi}{\lambda_h + 1} + \frac{1}{2} \right].
\]

Then, the relative load increase \( I_W \) of the present bearing due to the interfacial slippage, non-continuum and dynamic effects of the film, as compared to the conventional calculated load, is

\[
I_W = \frac{W_{bf,\text{slip}} - W_{\text{conv}}}{W_{\text{conv}}}.
\]

The dimensionless pressure in the ‘I’ sub zone is

\[
P_{\text{conv},I} = \frac{6X}{\alpha} \left( \frac{\psi + \lambda_h}{\lambda_h + \frac{\psi}{2} \left( 1 + \frac{1}{\lambda_h} \right)} - 1 \right) \quad \text{for} \quad 0 < X < \frac{1}{\psi + 1}.
\]

The dimensionless pressure in the ‘II’ sub zone is

\[
P_{\text{conv},II} = \frac{6}{k} \left[ \frac{\psi}{\lambda_h} - \frac{\lambda_h}{2\lambda_h + \psi} \left( 1 + \frac{1}{\lambda_h} \right) \right] \left( \frac{1}{\lambda_h^2} - \frac{1}{\lambda_h^2} \right)
\]

\[
\quad \text{for} \quad \frac{1}{\psi + 1} < X < 1.
\]

The dimensionless friction force per unit contact length on the upper contact surface in the bearing is

\[
\tilde{F}_{f,a,\text{conv}} = \frac{1}{\alpha(\psi + 1)} \left[ \frac{3(\psi + \lambda_h)}{\lambda_h} - 2 \right] + \frac{1}{k} \left[ 3 \left( \frac{\psi + \lambda_h}{\lambda_h + \frac{\psi}{2} \left( 1 + \frac{1}{\lambda_h} \right)} \right) - \frac{\psi + \lambda_h}{\lambda_h^2 + \frac{\psi\lambda h}{2} \left( 1 + \frac{1}{\lambda_h} \right)} \right] - 2 \ln \lambda_h.
\]

The dimensionless friction force per unit contact length on the lower contact surface in the bearing is

\[
\tilde{F}_{f,b,\text{conv}} = \frac{1}{\alpha(\psi + 1)} \left[ 4 - \frac{3(\psi + \lambda_h)}{\lambda_h + \frac{\psi}{2} \left( 1 + \frac{1}{\lambda_h} \right)} \right] +
\]

\[
\left( 3 \left( \frac{\psi + \lambda_h}{\lambda_h + \frac{\psi}{2} \left( 1 + \frac{1}{\lambda_h} \right)} \right) - \frac{\lambda_h}{\lambda_h} \right) - 2 \ln \lambda_h.
\]
\[ + \frac{1}{k} \left[ 4 \ln \lambda_h - 3(\psi + \lambda_h) \left( \frac{1}{\lambda_h + \frac{\psi}{2} \left( 1 + \frac{1}{\lambda_h} \right)} - \frac{1}{\lambda_h^2 + \frac{\psi \lambda_h}{2} \left( 1 + \frac{1}{\lambda_h} \right)} \right) \right] \]

The friction coefficients at the upper and lower contact surfaces in the bearing are respectively

\[ f_{a,\text{conv}} = \frac{\bar{F}_{f,a,\text{conv}}}{W_{\text{conv}}} \quad \text{and} \quad f_{b,\text{conv}} = \frac{\bar{F}_{f,b,\text{conv}}}{W_{\text{conv}}} \]

5 Calculation Results

Calculations were carried out for the pressure distributions, carried loads and friction coefficients of the bearing for different operational parameter values. In the calculation, the parameters \( C_{qI}(H_{m,I}) \) and \( C_{qII}(H_{m,II}) \) are expressed as the following general form \([16–18]\):

\[ C_{qI}(H_m) = \begin{cases} 1, & \text{for } H_m \geq 1 \\ b_0 + b_1 H_m + b_2 H_m^2 + b_3 H_m^3, & \text{for } 0 < H_m \leq 1 \end{cases} \]

where \( H_m \) is \( H_o \) or \( H_{m,II} \), and \( b_0, b_1, b_2 \) and \( b_3 \) are respectively constants.

The parameters \( C_{yI}(H_{m,I}) \) and \( C_{yII}(H_{m,II}) \) are expressed as the following general form \([16–18]\):

\[ C_{yI}(H_m) = \begin{cases} 1, & \text{for } H_m \geq 1 \\ a_0 + a_1/H_m + a_2/H_m^2, & \text{for } 0 < H_m \leq 1 \end{cases} \]

where \( H_m \) is \( H_o \) or \( H_{m,II} \), and \( a_0, a_1, \) and \( a_2 \) are respectively constants.

The parameters \( S_I(H_{m,I}) \) and \( S_{II}(H_{m,II}) \) are expressed as the following general form \([16–18]\):

\[ S(H_m) = \begin{cases} 1, & \text{for } H_m \geq 1 \\ \left[ n_0 + n_1(H_m - n_3)^{n_2} \right]^{-1}, & \text{for } 0 < H_m \leq 1 \end{cases} \]

where \( H_m \) is \( H_o \) or \( H_{m,II} \), and \( n_0, n_1, n_2 \) and \( n_3 \) are respectively constants.

There may be different interactions between the fluid and the stationary surface respectively in the ‘I’ and ‘II’ sub zones. The interaction between the fluid and the stationary surface in these two sub zones may be strong, medium-level or weak. In the calculation, it was taken that for the strong fluid-wall interaction in whichever
sub zone, \( h_{cr,bf} = 40 \) nm; for the medium-level interaction, \( h_{cr,bf} = 20 \) nm; for the weak interaction, \( h_{cr,bf} = 7 \) nm [16–18]. For modeling the different interactions between the fluid and the stationary surface, the values of the other parameters are respectively given in Tables 2a, 2b, 2c and 2d [16–18].

### Table 2a. Fluid viscosity data for different fluid-wall interactions [16–18]

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Parameter</th>
<th>( a_0 )</th>
<th>( a_1 )</th>
<th>( a_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td></td>
<td>1.8335</td>
<td>-1.4252</td>
<td>0.5917</td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td>1.0822</td>
<td>-0.1758</td>
<td>0.0936</td>
</tr>
<tr>
<td>Weak</td>
<td></td>
<td>0.9507</td>
<td>0.0492</td>
<td>1.6447E-4</td>
</tr>
</tbody>
</table>

### Table 2b. Fluid density data for different fluid-wall interactions [16–18]

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Parameter</th>
<th>( b_0 )</th>
<th>( b_1 )</th>
<th>( b_2 )</th>
<th>( b_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td></td>
<td>1.43</td>
<td>-1.723</td>
<td>2.641</td>
<td>1.347</td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td>1.30</td>
<td>-1.065</td>
<td>1.336</td>
<td>-0.571</td>
</tr>
<tr>
<td>Weak</td>
<td></td>
<td>1.116</td>
<td>-0.328</td>
<td>0.253</td>
<td>-0.041</td>
</tr>
</tbody>
</table>

### Table 2c. Fluid non-continuum data for different fluid-wall interactions [16–18]

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Parameter</th>
<th>( n_0 )</th>
<th>( n_1 )</th>
<th>( n_2 )</th>
<th>( n_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td></td>
<td>0.4</td>
<td>-1.374</td>
<td>-0.534</td>
<td>0.035</td>
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<tr>
<td>Medium</td>
<td></td>
<td>-0.649</td>
<td>-0.343</td>
<td>-0.665</td>
<td>0.035</td>
</tr>
<tr>
<td>Weak</td>
<td></td>
<td>-0.1</td>
<td>-0.892</td>
<td>-0.084</td>
<td>0.1</td>
</tr>
</tbody>
</table>

### Table 2d. Values of \( k_0 \) for different fluid-wall interactions [16–18]

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Parameter</th>
<th>( k_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strong</td>
<td></td>
<td>-0.536</td>
</tr>
<tr>
<td>Medium</td>
<td></td>
<td>-0.247</td>
</tr>
<tr>
<td>Weak</td>
<td></td>
<td>-0.1</td>
</tr>
</tbody>
</table>

### 6 Results and Discussion

Calculations were made for the studied bearing for varying operational parameter values, when \( \alpha = 2.5E-4 \), \( \psi = 1 \) and \( \lambda_h = 1.1 \). Results are discussed as follows.

#### 6.1 Pressure Distribution

Figures 2(a)–(c) respectively plot the dimensionless pressure distributions in the bearing for different \( \tilde{\tau}_{sa} \) values and \( h_0 = 3 \) nm for different fluid-wall interactions. The reduction of the interfacial shear strength \( \tilde{\tau}_{sa} \) is shown to significantly increase the pressure. All the pressures in the bearing are greatly higher than those calculated by conventional hydrodynamic lubrication theory. This is ascribed to the interfacial slippage, non-continuum and dynamic effects of the lubricating film as commented before [16–18].

Figure 2(d) gives the pressure distributions for different fluid-wall interactions when other operational parameter values are the same. It is shown that for inhomogeneous wall surfaces, the M-W interaction type is advantageous over the S-W
interaction type for generating the pressure, while the S-M interaction type appears the most advantageous.

6.2 Carried Load

Figures 3(a)–(c) plot the relative load increases $I_w$ of the bearing compared to the conventional hydrodynamic lubrication theory calculation against the dimensionless interfacial shear strength $\bar{\tau}_{sa}$ for different $h_o$ values and different fluid-wall interactions. It is shown that for a given $h_o$, $I_w$ is linearly increased with the reduction of $\bar{\tau}_{sa}$. This indicates that the designed boundary slippage significantly increases the load-carrying capacity of the bearing.
Figure 3 plots the relative load increase $I_w$ of the bearing for different fluid-wall interactions and different operating conditions, $\alpha = 2.5E-4$, $\psi = 1$ and $\lambda_h = 1.1$.

The reduction of $h_o$ increases the value of $I_w$, particularly for the S-M interaction type and low film thicknesses. This is ascribed to the strong non-continuum and dynamic effects of the lubricating film in such operating conditions.

6.3 THE RELATIVE SLIP AMOUNT

Figure 4 plots the distributions of the relative slip amount $\gamma_{s,II}$ in the bearing inlet zone for different fluid-wall interactions. The distribution of the value of $\gamma_{s,II}$ is linear. As can be seen from the value of $\gamma_{s,II}$, the S-W interaction type results in the strongest fluid-wall interfacial slippage, while the S-M interaction type generates the weakest interfacial slippage.
6.3 The relative slip amount

Figure 4 plots the distributions of the relative slip amount $\delta_s$, $\delta_J$ in the bearing inlet zone for different fluid-wall interactions. The distribution of the value of $\delta_s$, $\delta_J$ is linear. As can be seen from the value of $\delta_s$, $\delta_J$, the S-W interaction type results in the strongest fluid-wall interfacial slippage, while the S-M interaction type generates the weakest interfacial slippage.

Fig. 4. Plot of the relative slip amount in the bearing inlet zone for different fluid-wall interactions, $h_o = 3$ nm, $\bar{\tau}_{sa} = 0.1$, $\alpha = 2.5 \times 10^{-4}$, $\psi = 1$ and $\lambda_h = 1.1$.

6.4 Friction coefficient

Figures 5(a) and (b) respectively plot the friction coefficients on the upper and lower contact surfaces in the bearing against $\bar{\tau}_{sa}$ for different fluid-wall interactions when $h_o = 10$ nm. The reduction of $\bar{\tau}_{sa}$ considerably reduces the friction coefficients. The M-W interaction results in the lowest friction coefficients, while the S-W interaction gives the highest friction coefficients. The calculated friction coefficients of the bearing from the used model are much lower than those calculated from conventional hydrodynamic lubrication theory.

Fig. 5. Plots of the friction coefficients ($f_{a,b,\text{slip}}$, $f_{b,b,\text{slip}}$) of the bearing against $\bar{\tau}_{sa}$ for different fluid-wall interactions, $h_o = 10$ nm, $\alpha = 2.5 \times 10^{-4}$, $\psi = 1$, and $\lambda_h = 1.1$. 

0.00 0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16
0.000
0.002
0.004
0.006
0.008
$f_{a,b,\text{slip}}$, $f_{b,b,\text{slip}}$

0.00 0.02 0.04 0.06 0.08 0.10 0.12 0.14 0.16
0.000
0.002
0.004
0.006
0.008
$f_{a,b,\text{slip}}$, $f_{b,b,\text{slip}}$
7 CONCLUSIONS

This paper presents an analysis for a micro/nano wedge-platform thrust slider bearing by using the flow factor approach model, based on the fluid-wall interfacial shear strength. The fluid-wall interfacial slippage was designed at the stationary surface in the bearing inlet zone, while on the other bearing surfaces the interfacial slippage was prevented. The carried load and friction coefficient of the bearing were calculated. It was shown that the non-continuum and dynamic effects of the lubricating film and the designed interfacial slippage both have great contributions to the load-carrying capacity of the bearing: The S-M interaction type gives the highest load-carrying capacity of the bearing, while the S-W interaction type gives the lowest.

The friction coefficient of the bearing calculated from the model is much lower than that calculated from conventional hydrodynamic lubrication theory. The M-W interaction results in the lowest friction coefficient, while the S-W interaction gives the highest friction coefficient.

The value of the relative slip amount is linearly distributed in the bearing inlet zone. This value indicates that the S-W interaction results in the strongest fluid-wall interfacial slippage, while the S-M interaction generates the weakest interfacial slippage.

REFERENCES


