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DETERMINISTICALLY – PROBABILISTIC APPROACH FOR DETERMINING THE STEELS ELASTICITY MODULES*

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Abstract. The known deterministic relationships to estimate the elastic characteristics of materials are not well accounted for significant variability of these parameters in solids. Therefore, it is given a probabilistic approach to determine the modules of elasticity, adopted to random values, which increases the accuracy of the obtained results. By an ultrasonic testing, a non-destructive evaluation of the investigated steels structure and properties has been made.
Key words: Modulus of elasticity, deterministic and probabilistic approach, ultrasonic testing.

Introduction

The known deterministic relationships to estimate the elastic characteristics of materials are not well accounted for significant variability of these parameters in solids. Therefore, it is given a probabilistic approach to deter-

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mine the modules of elasticity, adopted to random values, which increases the accuracy of the obtained results. A non-destructive evaluation of the investigated steels structure and properties has been performed by an ultrasonic testing.

1. Deterministic approach

Lame constants of the material elastic properties characterization is used in the Theory of elasticity [1]. The easiest way to calculate their values, using an ultrasonic non- destructive testing, is:

$$\lambda + 2.\mu = \rho V_L^2; \quad \mu = \rho V_T^2, \tag{1}$$

where: V_L – longitudinal ultrasonic waves distribution velocity;
V_T – transverse ultrasonic waves distribution velocity;
ρ – density of material.

The velocity measurements are conducted according to a standard methodics [2, 3]. The modules of elasticity – Young's module E, Poisson's ratio υ, module of rigidity G and bulk module K are defined in Mechanics of Materials [4]. The relationship between the parameters, mentioned above, is:

$$\begin{aligned} &E = \left(3.\frac{\lambda}{\mu} + 2\right)\left(\frac{\lambda}{\mu} + 1\right)^{-1}\mu; \quad \nu = \frac{1}{2}\left(\frac{\lambda}{\mu}\right)\left(\frac{\lambda}{\mu} + 1\right)^{-1}; \\ &G = \mu; \quad K = \left(\frac{\lambda}{\mu} + \frac{2}{3}\right).\mu, \end{aligned} \tag{2}$$

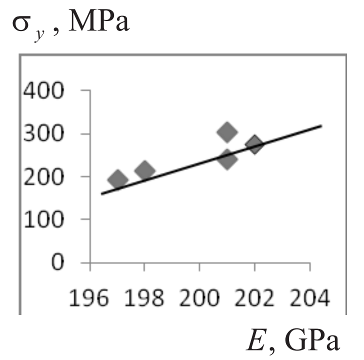
where: $\dfrac{\lambda}{\mu} = \left(\dfrac{V_L}{V_T}\right)^2 - 2$, $\mu = \rho V_T^2$.

2. Probabilistic approach

2.1. Statistical dependence $E - D$

A dependence $E - D$ has been output to give the probabilistic characteristics of (λ, μ) and (E, υ, G, K), where E is Young's module, but D is the average size of metal matrix grains. The following algorithm for low carbon steels is used:

Table 1

Model	
$\sigma_y = \gamma_0 + \gamma_1 E$ $\gamma_1 = 0.04048$ $\gamma_0 = 189.882$ $R \approx 0.85$	

a) According to data from [5] a relationship between yield point σ_y and Young's module E has been output – Table 1, where γ_0 and γ_1 are the model parameters;

b) The dependence of Hall- Petch $\sigma_y = \sigma_0 + K_y \left(\overline{D}\right)^{-1/2}$ [3, 6] is used. For low carbon steels:

$$\sigma_0 = 72 \text{ MPa}; \; K_y = 23.9 \text{ MPa.mm}^{1/2} \quad (R = 0.993), \; [6].$$

From a) and b) follows that $\sigma_0 + K_y \left(\overline{D}\right)^{-1/2} \equiv \gamma_0 + \gamma_1 E$ and, therefore:

$$E = \beta_0 + \beta_1 \left(\overline{D}\right)^{-1/2}, \tag{3}$$

where: $\beta_0 = (\sigma_0 - \gamma_0)/\gamma_1$; $\beta_1 = (K_y/\gamma_1)$. It can be concluded from the obtained results, that for carbon steels the average grain size D in (3) can be determined by an ultrasonic testing of its acoustic characteristics (V_L, V_T and α_L), [3].

$$\left[\frac{4.\pi^2}{1125}\frac{V_T^4}{V_L^3}\left(\frac{2}{V_L^5} + \frac{3}{V_T^5}\right).f\right]\left(\overline{D}\right)^3 - \alpha_L = 0. \tag{4}$$

2.2. Probabilistical characteristics of $(\boldsymbol{E})$, $(\boldsymbol{\lambda}; \boldsymbol{\mu})$ and $(\boldsymbol{G}; \boldsymbol{K})$

2.2.1. Density distribution, mathematical expectation and the random value $\boldsymbol{E}$ dispersion

To give the density distribution of the random value E, it is taken into account that in conducting a number of strength tests (empirical data) and from the normal law of distribution the random value σ_y is established.

Mechanical test results of a sample of more than 4000 specimens with different thickness are given in [3, 7], where for the random value σ_y, the normal distribution law is proven. From the above, the resulting model for σ_y (Table 1) and from the theorem for density distribution of functionally dependent random value of Probability Theory, follows that the random value E of carbon steels also have normal distribution with density $p_{Gauss}(E)$, which is analytically written as [7, 8]:

$$p_{Gauss}(E) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(E-a)^2}{2\sigma^2}\right), \tag{5}$$

where: a and σ are distribution parameters.

One approach to the experimental determination of a and σ is through the use of dependence (3). Since for the average grain size of the steels in the condition $\left(\overline{D}\right)^2 < 1$, the function $\left(\overline{D}\right)^{-1/2}$ (3) may occur in a power series. After rejecting the terms in the order of $\left(\overline{D}\right)^3$ in it, the approximate dependence $\left(\overline{D}\right)^{-1/2} \approx \frac{15}{8} - \frac{5}{4}\overline{D} + \frac{3}{8}\left(\overline{D}\right)^2$ is obtained and therefore: $E \approx \beta_0 + \beta_1 \left[\frac{15}{8} - \frac{5}{4}\overline{D} + \frac{3}{8}\left(\overline{D}\right)^2\right]$. The D is evaluated by measuring of V_L, V_T and α_L of (4), than for $M(E)$ – mathematical expectation and $D(E)$ – dispersion can be written:

$$M(E) \approx \left(\beta_0 + \frac{15}{8}\beta_1\right) - \left(\frac{5}{4}\beta_1\right)\overline{D} + \left(\frac{3}{8}\beta_1\right)\left(\overline{D}\right)^2, \tag{6}$$

$$D(E) \approx \beta_0^2 \left(\overline{D}\right)^{-1} - \beta_1^2 \left\{1.125 + 0.5\overline{D} - 0.375\left(\overline{D}\right)^2\right\}^{-2} \tag{7}$$

i.e. $M(E) \approx a(D)$ and $D(E) \approx \sigma(D)$, and therefore:

$$a = a\left(\overline{D}\right) \quad \text{and} \quad \sigma^2 = \sigma\left(\overline{D}\right). \tag{8}$$

2.2.2. Density distribution, mathematical expectation and the random values $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ dispersion

Consider dependencies:

$$\lambda = a_\lambda(\nu)E; \quad \mu = a_\mu(\nu)E, \tag{9}$$

where: $a_\lambda(\nu) = \dfrac{\nu}{(1+\nu)(1-2.\nu)}$, $a_\mu(\nu) = \dfrac{1}{2.(1+\nu)}$.

As for carbon steels, the Poisson's ratio is written in a relatively narrow range, namely $0.23 \leq \upsilon \leq 0.32$, it can be considered an average value of $\upsilon \approx 0.28$. In this sense, in the parameters $a_\lambda(\nu)$ and $a_\mu(\nu)$ from (9) are constants, i.e. $a_\lambda(\nu) \equiv a_\lambda$ and $a_\mu(\nu) \equiv a_\mu$.

Density distributions $p(\lambda)$ and $p(\mu)$ of the random values λ and μ are obtained from the theorem of density functional distribution of dependent random value (Probability Theory) and applied to the dependencies:

$$p(\lambda) = \frac{1}{a_\lambda} p_{Gauss}(E)\,; \quad p(\mu) = \frac{1}{a_\mu} p_{Gauss}(E)\,. \tag{10}$$

The mathematical expectation and the dispersion of the random values λ and μ are given in:

$$\begin{aligned} &M(\lambda) = a_\lambda M(E); \quad M(\mu) = a_\mu M(E) \quad \text{and} \\ &D(\lambda) = a_\lambda^2 D(E); \quad D(\mu) = a_\mu^2 D(E). \end{aligned} \tag{11}$$

2.2.3. Density distribution, mathematical expectation and the random values *G* and *K* dispersion

The density of the distribution $p(G)$, the mathematical expectation $M(G)$ and the dispersion $D(G)$ of the random value G are obtained taking into account that $G \equiv \mu$, i.e. $p(G) \equiv p(\mu)$, and therefore:

$$p(G) = \frac{1}{a_\mu} p_{Gauss}(E)\,. \tag{12}$$

The density of distribution $p(K)$, the mathematical expectation $M(K)$ and the dispersion $D(K)$ of the random value K are obtained taking into account that $K = a_k.E$, where: $a_K = \dfrac{1}{3(1-2\nu)}$ [1] and therefore:

$$\begin{aligned} &p(K) = \frac{1}{a_K} p_{Gauss}(E)\,; \\ &M(K) = a_K M(E) \quad \text{and} \\ &D(K) = a_K^2 D(E). \end{aligned} \tag{13}$$

3. Experimental results

Specimens are taken of low carbon construction steels that match the specification of AISI1006 to AISI1095. About them, the Poisson's ratio υ [5] is in the range of $0.23 \leq \upsilon \leq 0.32$.

The following technical devices are used: ultrasonic flow detector with an option "time measuring of ultrasonic waves propagation", with an accuracy of 0.01 μs; ultrasonic sensors with X-cut (longitudinal waves and Y-cut (transverse waves) at a frequency of 5 MHz; micrometer MITUTOYO – Japan, with accuracy 0.5 μm.

Propagation velocities of ultrasonic waves are measured:

$$V_L = (5.92 \pm 0.030)\ \text{mm}/\mu\text{s} \quad \text{and} \quad V_T = (3.255 \pm 0.015)\ \text{mm}/\mu\text{s}.$$

For carbon steels it is assumed that $\rho = 7.85$ g.cm^{-3}.

The elasticity modulus μ, λ, E, υ, G and K are determined.

From (1), follows that: $\lambda = \rho V_L^2 - 2.\rho V_T^2$; $\mu = \rho V_T^2 \Rightarrow \lambda = 108.08$ GPa; $\mu = 82.64$ GPa.

As it is known, the numerical values of the modulus, determined by ultrasonic measurements, are different. This difference is of the order of: 3–5%, i.e. $\lambda_M = \lambda_U/\Delta$ and $\mu_M = \mu_U/\Delta$; $\Delta = 1.03 - 1.05$. Therefore: $\lambda = 104.9$ GPa and $\mu = 80.2$ GPa.

As designated λ and μ, E, υ, G and K are calculated. The results are shown in Table 2:

Table 2

Theoretical values of modules for low carbon steels [5]	
$(\lambda;\mu)$=(xxx; 80.1 GPa)	$(E;\nu;G;K)$=(202 GPa; 0.261; 80.1 GPa; xxx)
Experimental results for	
$(\lambda_U;\mu_U)$=(104.9 GPa; 80.2 GPa)	$(E;\nu;G;K)$=(201 GPa; 0.28; 80.2 GPa; 158 GPa)

Density distributions, mathematical expectations and dispersions $M(\lambda)$, $M(\mu)$, $M(K)$ and $D(\lambda)$, $D(\mu)$, $D(G)$, $D(K)$ are explicitly specified.

The results are presented in Table 3:

The parameters a and σ^2 are determined by the average grain size of carbon steels. The value D is determined by measuring the acoustic characteristics of material [3].

Table 3

Density distributions	Mathematical expectations	Dispersions
$p(\lambda) = 2.012 p_{Gauss}(E)$	$M(\lambda) = 0.497 M(E)$	$D(\lambda) = 0.247 D(E)$
$p(\mu) = 2.557 p_{Gauss}(E)$	$M(\mu) = 0.391 M(E)$	$D(\mu) = 0.153 D(E)$
$p(K) = 1.32 p_{Gauss}(E)$	$M(K) = 0.758 M(E)$	$D(K) = 0.575 D(E)$

where: $P_{Gauss}(E)$ is given in (4).

Conclusion

The article contains the following results, which are new:

1. A deterministic approach for non-destructive evaluation of the Lame constants $(\lambda,\ \mu)$ and the modules of elasticity (E, υ, G, K) is considered by measuring the acoustic characteristics of the material.
2. For carbon steels, derived dependencies are $E - D$ and $\sigma_y - E$.
3. It is proved that the random variable for carbon steels has a normal distribution with density $P_{Gauss}(E)$. Relationships are derived for the parameters of $P_{Gauss}(E)$, i.e. $a = a(\overline{D})$ and $\sigma^2 = \sigma(\overline{D})$, the assessrment of the average grain size D is performed by measuring the acoustic characteristics $(VL, VT, \alpha L)$ of the material.
4. For the random values (λ, μ) and (G, K) density distributions: $p(\lambda)$, $p(\mu)$, $p(G)$, $p(K)$, as well as the mathematical expectations: $M(\lambda)$, $M(\mu)$, $M(G)$, $M(K)$ and dispersion $D(\lambda)$, $D(\mu)$, $D(G)$, $D(K)$ are derived.

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