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PROPAGATION OF SURFACE WAVES IN A HOMOGENEOUS LAYER OF FINITE THICKNESS OVER AN INITIALLY STRESSED FUNCTIONALLY GRADED MAGNETIC-ELECTRIC-ELASTIC HALF-SPACE*

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Abstract. The propagation behaviour of Love wave in an initially stressed functionally graded magnetic-electric-elastic half-space carrying a homogeneous layer is investigated. The material parameters in the substrate are assumed to vary exponentially along the thickness direction only. The velocity equations of Love wave are derived on the electrically or magnetically open circuit and short circuit boundary conditions, based on the equations of motion of the graded magnetic-electric-elastic material with the initial stresses and the free traction boundary conditions of surface and the continuous boundary conditions of interface. The dispersive curves are obtained numerically and the influences of the initial stresses and the material gradient index on the dispersive curves are discussed. The investigation provides a basis for the development of new functionally graded magneto-electro-elastic surface wave devices.
Key words: Initial stress, graded material; magnetic-electric-elastic material; surface wave; open and short circuit.

1. Introduction

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The magnetic-electric-elastic materials possess particular product properties, i.e. the magneto-electric-mechanical coupling effect, which is not demonstrated with conventional piezoelectric or piezomagnetic materials. The mechanics of the magnetic-electric-elastic materials has received considerable research effort with their increasing usage in various applications including sensors and actuators [1, 2]. These applications are closely related to vibration and wave propagation properties of the magnetic-electric-elastic materials. So, the research work of the dynamic properties about this kind of structure attracts much attention in recent years. Calas et al. [3] investigated the propagation of shear horizontal (SH) waves in magnetic-electric-elastic multilayered structures. Liu et al. [4] studied the Love wave in piezomagnetic and piezoelectric structures. Feng et al. [5] and Li et al. [6] considered Rayleigh waves in magnetic-electric-elastic half planes. Wu et al. [7] studied the Lame waves in the piezoelectric-piezomagnetic bimaterial. Functionally graded materials have extensive applications in many fields, such as aerospace, electronics. The increasing utilization of the functionally graded materials has required better understanding of their mechanical and thermal behaviour. As for the wave propagation problem, although numerous achievements have been made for functionally graded piezoelectric materials [8–12], research on the wave propagation in functionally graded magnetic-electric-elastic materials is still very limited. Chen et al. [13] considered the free vibration problem of the functionally graded magnetic-electric-elastic multilayered plates. Chen et al. [14] investigated propagation of axial shear magneto-electro-elastic waves in piezoelectric-piezomagnetic composites with randomly distributed cylindrical inhomogeneities. Wu et al. [15] investigated the harmonic waves in inhomogeneous magnetic-electric-elastic plates. Yu et al. [16–17] studied the wave propagation in various inhomogeneous curved waveguides. In all the abovementioned researches, the initial stresses induced during the processing were not taken into account. However, for the FGM piezomagnetic-piezoelectric structure, due to the non-uniform material properties, coefficients of thermal expansion and chemical/nucleation shrinkage/growth during the processing, the presence of initial stress is unavoidable. It is necessary to investigate the effects of the initial stresses on the wave propagation behaviour in functionally graded magnetic-electric-elastic materials and structures. Li et al. [18] investigated the propagation behaviour of shear surface wave in a functionally graded magneto-electro-elastic half-space with initial stresses and discussed the effects of the initial stresses and material gradient index on the surface wave velocities.

In this paper, we have taken into account the effects of initial stresses on the propagation behaviour in a functionally graded magnetic-electric-elastic half-space, carrying a homogeneous layer and discuss the effects of the initial stresses and the material gradient index on the surface wave velocities. For convenience in the analysis, we assume that material properties change exponentially along the thickness direction in the substrate. The speed equations of Love wave are derived under the initial stresses on different electrically and magnetically boundary conditions. Some significant results have been obtained, which can provide a theoretical foundation for the design and practical application of surface acoustic wave devices with the functionally graded magneto-electro-elastic structures.

2. Statement of the problem

Here, the wave propagation behaviour in a homogeneous layered functionally graded magneto-electro-elastic structure is taken into account, as shown in Fig. 1. It involves an isotropic homogenous layer with uniform thickness of h bonded perfectly to a transversely isotropic magneto-electro-elastic substrate with its polarization direction perpendicular to the x-y plane. It is assumed, that the wave propagation is in the positive direction of x axis and constant initial stresses are in the substrate.

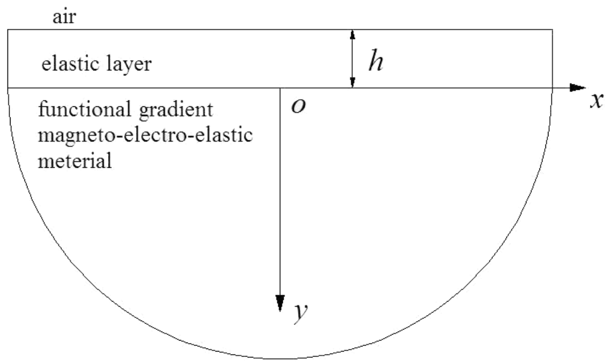


Fig. 1. Elastic layered functionally graded magnetic-electric-elastic half space

2.1. Initially stressed functionally graded magnetic-electric-elastic half space

The constitutive equations of the magnetic-electric-elastic solid can be written as:

$$\sigma_{ij} = c_{ijkl}u_{k,l} + e_{kij}\varphi_{,k} + h_{kij}\phi_{,k}, \tag{1a}$$

$$D_i = e_{ikl}u_{k,l} - \kappa_{ik}\varphi_{,k} - \beta_{ik}\phi_{,k}, \tag{1b}$$

$$B_i = h_{ikl}u_{k,l} - \beta_{ik}\varphi_{,k} - \mu_{ik}\phi_{,k}. \tag{1c}$$

For the wave motion of small amplitude, the equations of motion of the magnetic-electric-elastic solid with initial stresses can be written as [19]:

$$\sigma_{ji,j} + (u_{i,k}\sigma^0_{kj})_{,j} = \rho\ddot{u}_i \ \ , \ D_{i,i} = 0 \ \ , \ B_{i,i} = 0 \tag{2}$$

In Eqs (1)–(2), $\sigma_{ij}$is the stress tensor; σ^0_{kj} is the initial stress; The coefficients c_{ijkl}, κ_{ik} and $\mu_{ik}$are the elastic constant, dielectric constant and the magnetic permittivity, respectively. The coefficientse_{kij}, $h_{kij}$and β_{ik} are the piezoelectric, piezomagnetic and electromagnetic constant, respectively. u_i is the mechanical displacement vector, D_i the electric displacement vector, B_i the magnetic induction vector. φ, ϕ are the electric and magnetic potential, respectively, ρ the mass density.

For the transversely isotropic magneto-electro-elastic substrate, the constitutive equations can be written in term of components:

$$\begin{aligned}
&\sigma_{xx} = c_{11}\varepsilon_{xx} + c_{12}\varepsilon_{yy} + c_{13}\varepsilon_{zz} + e_{31}\varphi_{,z} + h_{31}\phi_{,z},\\
&\sigma_{yy} = c_{12}\varepsilon_{xx} + c_{11}\varepsilon_{yy} + c_{13}\varepsilon_{zz} + e_{31}\varphi_{,z} + h_{31}\phi_{,z},\\
&\sigma_{zz} = c_{13}\varepsilon_{xx} + c_{13}\varepsilon_{yy} + c_{33}\varepsilon_{zz} + e_{33}\varphi_{,z} + h_{33}\phi_{,z},\\
&\sigma_{yz} = 2c_{44}\varepsilon_{yz} + e_{15}\varphi_{,y} + h_{15}\phi_{,y},\\
&\sigma_{zx} = 2c_{44}\varepsilon_{zx} + e_{15}\varphi_{,x} + h_{15}\phi_{,x},\\
&\sigma_{xy} = (c_{11} - c_{12})\varepsilon_{xy},\\
&D_x = 2e_{15}\varepsilon_{zx} - \kappa_{11}\varphi_{,x} - \beta_{11}\phi_{,x},\\
&D_y = 2e_{15}\varepsilon_{yz} - \kappa_{11}\varphi_{,y} - \beta_{11}\phi_{,y},\\
&D_z = e_{31}\varepsilon_{xx} + e_{31}\varepsilon_{yy} + e_{33}\varepsilon_{zz} - \kappa_{33}\varphi_{,z} - \beta_{33}\phi_{,z},\\
&B_x = 2h_{15}\varepsilon_{zx} - \beta_{11}\varphi_{,x} - \mu_{11}\phi_{,x},\\
&B_y = 2h_{15}\varepsilon_{yz} - \beta_{11}\varphi_{,y} - \mu_{11}\phi_{,y},\\
&B_z = h_{31}\varepsilon_{xx} + h_{31}\varepsilon_{yy} + h_{33}\varepsilon_{zz} - \beta_{33}\varphi_{,z} - \mu_{33}\phi_{,z}.
\end{aligned} \tag{3}$$

The position dependent material characteristics and the initial stresses are assumed to vary exponentially along the thickness direction, i.e.:

$$\begin{aligned}
&c_{ik}(y) = c^0_{ik}e^{ky} \ \ , \ e_{ik}(y) = e^0_{ik}e^{ky},\\
&h_{ik}(y) = h^0_{ik}e^{ky} \ \ , \ \kappa_{ik}(y) = \kappa^0_{ik}e^{ky},\\
&\beta_{ik}(y) = \beta^0_{ik}e^{ky} \ \ , \ \mu_{ik}(y) = \mu^0_{ik}e^{ky},\\
&\rho = \rho^0 e^{ky} \ \ , \ \sigma^0_{kj}(y) = \sigma^0_{kj0}e^{ky},
\end{aligned} \tag{4}$$

where $c_{ik}^0 = c_{ik}(0)$, $e_{ik}^0 = e_{ik}(0)$, $h_{ik}^0 = h_{ik}(0)$, $\kappa_{ik}^0 = \kappa_{ik}(0)$, $\beta_{ik}^0 = \beta_{ik}(0)$, $\mu_{ik}^0 = \mu_{ik}(0)$, $\rho^0 = \rho(0)$ and $\sigma_{kj0}^0 = \sigma_{kj}^0(0)$. k is the functional gradient index.

For the Love wave propagating in the positive direction of x axis, the mechanical displacement components and the electric and the magnetic potential are as following:

$$\text{(5)} \qquad u(x,y) = v(x,y) = 0, \quad w = w(x,y,t),$$
$$\varphi = \varphi(x,y,t), \quad \phi = \phi(x,y,t).$$

Substituting Eq.(3)–Eq.(5) into Eq.(2), only consider the initial stresses σ_{xx}^0, σ_{yy}^0in the substrate, we have the following equations of motion:

$$\text{(6a)} \quad c_{44}^0 \nabla^2 w + e_{15}^0 \nabla^2 \varphi + h_{15}^0 \nabla^2 \phi + k \left(c_{44}^0 \frac{\partial w}{\partial y} + e_{15}^0 \frac{\partial \varphi}{\partial y} + h_{15}^0 \frac{\partial \phi}{\partial y} \right) = \rho^0 \frac{\partial^2 w}{\partial t^2} - \sigma_1^0 \frac{\partial^2 w}{\partial x^2} - \sigma_2^0 \frac{\partial^2 w}{\partial y^2},$$

$$\text{(6b)} \quad e_{15}^0 \nabla^2 w - \kappa_{11}^0 \nabla^2 \varphi - \beta_{11}^0 \nabla^2 \phi + k \left(e_{15}^0 \frac{\partial w}{\partial y} - \kappa_{11}^0 \frac{\partial \varphi}{\partial y} - \beta_{11}^0 \frac{\partial \phi}{\partial y} \right) = 0,$$

$$\text{(6c)} \quad h_{15}^0 \nabla^2 w - \beta_{11}^0 \nabla^2 \varphi - \mu_{11}^0 \nabla^2 \phi + k \left(h_{15}^0 \frac{\partial w}{\partial y} - \beta_{11}^0 \frac{\partial \varphi}{\partial y} - \mu_{11}^0 \frac{\partial \phi}{\partial y} \right) = 0,$$

where $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$, $\sigma_1^0 = \sigma_{xx0}^0$, $\sigma_2^0 = \sigma_{yy0}^0$. Introduce the two functions:

$$\text{(7)} \qquad \psi = \varphi - mw, \quad \chi = \phi - nw.$$

Substitution of Eq.(7) into Eq.(6) yields:

$$\text{(8a)} \qquad \bar{c}_{44}^0 \left(\nabla^2 w + k \frac{\partial w}{\partial y} \right) = \rho^0 \frac{\partial^2 w}{\partial t^2} - \sigma_1^0 \frac{\partial^2 w}{\partial x^2} - \sigma_2^0 \frac{\partial^2 w}{\partial y^2},$$

$$\text{(8b)} \qquad \nabla^2 \psi + k \frac{\partial \psi}{\partial y} = 0,$$

$$\text{(8c)} \qquad \nabla^2 \chi + k \frac{\partial \chi}{\partial y} = 0.$$

In Eqs. (7)–(8):

$$m = \frac{\mu_{11}^0 e_{15}^0 - \beta_{11}^0 h_{15}^0}{\kappa_{11}^0 \mu_{11}^0 - (\beta_{11}^0)^2}, \quad n = \frac{\kappa_{11}^0 h_{15}^0 - \beta_{11}^0 e_{15}^0}{\kappa_{11}^0 \mu_{11}^0 - (\beta_{11}^0)^2}, \tag{9}$$

$$\bar{c}_{44}^0 = c_{44}^0 + \frac{[\mu_{11}^0 (e_{15}^0)^2 + \kappa_{11}^0 (h_{15}^0)^2 - 2\beta_{11}^0 e_{15}^0 h_{15}^0]}{[\kappa_{11}^0 \mu_{11}^0 - (\beta_{11}^0)^2]}. \tag{10}$$

Then, the stress tensor, electric displacement vector and the magnetic induction vector in Eqs. (3) can be expressed in term of w, ψ and χ:

$$\begin{aligned}
&\sigma_{xx} = \sigma_{yy} = \sigma_{zz} = \sigma_{xy} = 0, \quad D_z = 0, \quad B_z = 0,\\
&\sigma_{yz} = e^{ky}\left(\bar{c}_{44}^0 \frac{\partial w}{\partial y} + e_{15}^0 \frac{\partial \psi}{\partial y} + h_{15}^0 \frac{\partial \chi}{\partial y}\right),\\
&\sigma_{xz} = e^{ky}\left(\bar{c}_{44}^0 \frac{\partial w}{\partial x} + e_{15}^0 \frac{\partial \psi}{\partial x} + h_{15}^0 \frac{\partial \chi}{\partial x}\right),\\
&D_x = e^{ky}\left(-\kappa_{11}^0 \frac{\partial \psi}{\partial x} - \beta_{11}^0 \frac{\partial \chi}{\partial x}\right),\\
&D_y = e^{ky}\left(-\kappa_{11}^0 \frac{\partial \psi}{\partial y} - \beta_{11}^0 \frac{\partial \chi}{\partial y}\right),\\
&B_x = e^{ky}\left(-\beta_{11}^0 \frac{\partial \psi}{\partial x} - \mu_{11}^0 \frac{\partial \chi}{\partial x}\right),\\
&B_y = e^{ky}\left(-\beta_{11}^0 \frac{\partial \psi}{\partial y} - \mu_{11}^0 \frac{\partial \chi}{\partial y}\right).
\end{aligned} \tag{11}$$

2.2. The homogeneous elastic layer

Let w', φ' and ϕ' denote the mechanical displacement, electric and magnetic potential in the homogeneous elastic layer. For the isotropic homogeneous layer, considering Eqs. (8) and Eqs. (11), we have the following equations of motion:

$$\mu'(\nabla^2 w') = \rho' \frac{\partial^2 w'}{\partial t^2}, \tag{12a}$$

$$\nabla^2 \varphi' = 0, \tag{12b}$$

$$\nabla^2 \phi' = 0, \tag{12c}$$

and the components of σ'_{ij}, D'_i and B'_i in the homogeneous elastic layer are:

$$(13)\qquad \begin{aligned} &\sigma'_{xx} = \sigma'_{yy} = \sigma'_{zz} = \sigma'_{xy} = 0, \quad D'_z = 0, \quad B'_z = 0,\\ &\sigma'_{yz} = \mu' \frac{\partial w'}{\partial y},\\ &\sigma'_{xz} = \mu' \frac{\partial w'}{\partial x},\\ &D'_x = -\kappa'_{11} \frac{\partial \varphi'}{\partial x},\\ &D'_y = -\kappa'_{11} \frac{\partial \varphi'}{\partial y},\\ &B'_x = -\mu'_{11} \frac{\partial \phi'}{\partial x},\\ &B'_y = -\mu'_{11} \frac{\partial \phi'}{\partial y}, \end{aligned}$$

where: the coefficients ρ', μ', κ'_{11} and μ'_{11} are the mass density, shear modulus, dielectric constant and the magnetic permittivity, respectively in the homogeneous elastic layer.

3. The velocity equation of Love wave

Let the area $y \geq 0$ is the functional gradient magnetic-electric-elastic material, marked as A. Let φ_A and ϕ_A denote the electric and magnetic potential, D_A and B_A the electric displacement vector and the magnetic induction vector along y in the region A. The stressσ_{yz} is marked asσ_A.

For $y \to +\infty$, $w = 0$, $\varphi_A = 0$, $\phi_A = 0$. The solution of Eq. (8) can be assumed as:

$$(14a)\qquad w = A_1 e^{-\eta y} e^{i(\xi x - \omega t)},$$

$$(14b)\qquad \psi = A_2 e^{-\zeta y} e^{i(\xi x - \omega t)},$$

$$(14c)\qquad \chi = A_3 e^{-\zeta y} e^{i(\xi x - \omega t)},$$

where: A_1, A_2 and A_3 are unknown constants, ξ and ω the wave number and angular frequency, respectively. Substitution of Eq.(14) into Eq.(11) yields:

$$\begin{aligned}
&\sigma_A = -e^{ky}(\bar{c}_{44}^0\eta A_1 e^{-\eta y} + e_{15}^0\zeta A_2 e^{-\zeta y} + h_{15}^0\zeta A_3 e^{-\zeta y})e^{i(\xi x-\omega t)},\\
&D_A = e^{ky}(\kappa_{11}^0\zeta A_2 e^{-\zeta y} + \beta_{11}^0\zeta A_3 e^{-\zeta y})e^{i(\xi x-\omega t)},\\
&B_A = e^{ky}(\beta_{11}^0\zeta A_2 e^{-\zeta y} + \mu_{11}^0\zeta A_3 e^{-\zeta y})e^{i(\xi x-\omega t)},\\
&\varphi_A = (A_2 e^{-\zeta y} + mA_1 e^{-\eta y})e^{i(\xi x-\omega t)},\\
&\phi_A = (A_3 e^{-\zeta y} + nA_1 e^{-\eta y})e^{i(\xi x-\omega t)}.
\end{aligned} \tag{15}$$

Substituting Eq.(14) into Eq.(8), for $v < v_A$, we have:

$$(1+\frac{\sigma_2^0}{\bar{c}_{44}^0})\eta^2 - k\eta = \xi^2(1-\frac{v^2}{v_A^2}+\frac{\sigma_1^0}{\bar{c}_{44}^0}) > 0, \tag{16a}$$

$$\zeta^2 - \zeta k = \xi^2. \tag{16b}$$

Then the solution of η, ζ can be obtained from Eq.(16) as:

$$\eta = \frac{k+\sqrt{k^2+4(1+\frac{\sigma_2^0}{\bar{c}_{44}^0})(1-\frac{v^2}{v_A^2}+\frac{\sigma_1^0}{\bar{c}_{44}^0})\xi^2}}{2(1+\frac{\sigma_2^0}{\bar{c}_{44}^0})} > 0, \tag{17a}$$

$$\zeta = \frac{k+\sqrt{k^2+4\xi^2}}{2}, \tag{17b}$$

where: $v = \omega/\xi$ is the surface wave velocity and $v_A^2 = \bar{c}_{44}^0/\rho^0$.

The solution of Eq. (12) can be assumed as:

$$w' = (B_1 e^{-i\eta' y} + B_2 e^{i\eta' y})e^{i(\xi x-\omega t)}, \tag{18a}$$

$$\varphi' = (B_3 e^{-\xi y} + B_4 e^{\xi y})e^{i(\xi x-\omega t)}, \tag{18b}$$

$$\phi' = (B_5 e^{-\xi y} + B_6 e^{\xi y})e^{i(\xi x-\omega t)}, \tag{18c}$$

where: B_1, B_2, $\ldots$, B_6 are unknown constants.

Let the area $-h < y \leq 0$ is the homogeneous elastic layer, marked as B. Letφ_B and ϕ_B denote the electric and magnetic potential, D_B and B_B the electric displacement vector and the magnetic induction vector along y in the region B, respectively. The stress σ'_{yz} is marked as σ_B. Substitution of Eq.(18) into Eq.(13) yields:

$$\begin{aligned}
\sigma_B &= i\mu'\eta'(-B_1 e^{-i\eta' y} + B_2 e^{i\eta' y})e^{i(\xi x-\omega t)},\\
D_B &= (B_3\kappa'_{11}\xi e^{-\xi y} - B_4\kappa'_{11}\xi e^{\xi y})e^{i(\xi x-\omega t)},\\
B_B &= (B_5\mu'_{11}\xi e^{-\xi y} - B_6\mu'_{11}\xi e^{\xi y})e^{i(\xi x-\omega t)},\\
\varphi_B &= (B_3 e^{-\xi y} + B_4 e^{\xi y})e^{i(\xi x-\omega t)},\\
\phi_B &= (B_5 e^{-\xi y} + B_6 e^{\xi y})e^{i(\xi x-\omega t)}.
\end{aligned} \tag{19}$$

Substituting Eq.(18) into Eq.(12), we have:

$$\eta' = \sqrt{\frac{v^2}{v_B^2} - 1}\xi, \tag{20}$$

where: $v_B^2 = \mu'/\rho'$.

In vacuum area C, the electric potential φ_C and magnetic potential ϕ_C satisfie Laplace's equations, i.e.:

$$\nabla^2\varphi_C = 0, \quad \nabla^2\phi_C = 0. \tag{21}$$

For $y \to -\infty$, $\varphi_C = 0$, $\phi_C = 0$. The solution of Eq. (21) can be assumed to possess the following form:

$$\varphi_C = C_1 e^{\xi y} e^{i(\xi x-\omega t)}, \tag{22a}$$

$$\phi_C = C_2 e^{\xi y} e^{i(\xi x-\omega t)}, \tag{22b}$$

where: C_1 and C_2 are unknown constants. In vacuum, the electric displacement vector and the magnetic induction vector are expressed as, respectively:

$$D_C = \kappa_0 E_C = -\kappa_0 \frac{\partial\varphi_C}{\partial y} = -\kappa_0 C_1 e^{\xi y}\xi e^{i(\xi x-\omega t)}, \tag{23a}$$

$$B_C = \mu_0 H_C = -\mu_0 \frac{\partial\phi_C}{\partial y} = -\mu_0 C_2 e^{\xi y}\xi e^{i(\xi x-\omega t)}, \tag{23b}$$

where: $\kappa_0 = 8.854 \times 10^{-12} C^2 \cdot N^{-1} \cdot m^{-1}$ is the dielectric constant and $\mu_0 = 4\pi \times 10^{-7} Ns^2 \cdot C^{-2}$ is the magnetic permittivity in vacuum.

The following boundary and continuous conditions should be satisfied, when the surface wave propagates in the layered structure, as shown in Fig. 1. It should be pointed out that two kinds of magneto-electro boundary conditions, i. e. magneto-electro open and short conditions, would be taken into account in this study.

The mechanical traction-free, magnetically and electrically short circuit conditions at $y = -h$ and the continuous conditions at $y = 0$ satisfy:

$$
\begin{aligned}
&\sigma_B(x,-h,t) = 0, && \varphi_B(x,-h,t) = 0, \quad \phi_B(x,-h,t) = 0,\\
&\sigma_A(x,0,t) = \sigma_B(x,0,t), && w(x,0,t) = w'(x,0,t),\\
&\varphi_A(x,0,t) = \varphi_B(x,0,t), && D_A(x,0,t) = D_B(x,0,t),\\
&\phi_A(x,0,t) = \phi_B(x,0,t), && B_A(x,0,t) = B_B(x,0,t).
\end{aligned} \tag{24}
$$

which results in the algebraic equations in the unknowns A_1, A_2, A_3, B_1, B_2, B_3, B_4, B_5, B_6:

$$
\begin{aligned}
&-e^{i\eta' h} B_1 + e^{-i\eta' h} B_2 = 0\\
&e^{\xi h} B_3 + e^{-\xi h} B_4 = 0\\
&e^{\xi h} B_5 + e^{-\xi h} B_6 = 0\\
&-(\bar{c}^0_{44}\eta A_1 + e^0_{15}\zeta A_2 + h^0_{15}\zeta A_3) = i\mu'\eta'(-B_1 + B_2),\\
&A_1 = B_1 + B_2,\\
&mA_1 + A_2 = B_3 + B_4,\\
&\kappa^0_{11}\zeta A_2 + \beta^0_{11}\zeta A_3 = \kappa'_{11}\xi B_3 - \kappa'_{11}\xi B_4,\\
&nA_1 + A_3 = B_5 + B_6,\\
&\beta^0_{11}\zeta A_2 + \mu^0_{11}\zeta A_3 = \mu'_{11}\xi B_5 - \mu'_{11}\xi B_6.
\end{aligned} \tag{25}
$$

The nontrivial solution of A_1, A_2, A_3, B_1, B_2, B_3, B_4, B_5, B_6 exists only if the determinants of the coefficient matrix of Eq.(25) equals to zero, i.e.:

$$\begin{vmatrix} 0 & 0 & 0 & -e^{i\eta' h} & e^{-i\eta' h} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\xi h} & e^{-\xi h} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\xi h} & e^{-\xi h} \\ -\bar{c}^0_{44}\eta & -e^0_{15}\zeta & -h^0_{15}\zeta & i\mu'\eta' & -i\mu'\eta' & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\ m & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & \kappa^0_{11}\zeta & \beta^0_{11}\zeta & 0 & 0 & -\kappa'_{11}\xi & \kappa'_{11}\xi & 0 & 0 \\ n & 0 & 1 & 0 & 0 & 0 & 0 & -1 & -1 \\ 0 & \beta^0_{11}\zeta & \mu^0_{11}\zeta & 0 & 0 & 0 & 0 & -\mu'_{11}\xi & \mu'_{11}\xi \end{vmatrix} = 0. \tag{26}$$

from Eq.(26) , we have:

$$\begin{aligned} \bar{c}^0_{44}\eta - \mu'\eta'\tan(\eta' h) = \Big(& (m\kappa'_{11}\mu'_{11}e^0_{15} + n\kappa'_{11}\mu'_{11}h^0_{15})\xi^2\zeta \\ & + (m\kappa'_{11}\mu^0_{11}e^0_{15} + n\mu'_{11}\kappa^0_{11}h^0_{15} - m\kappa'_{11}\beta^0_{11}h^0_{15} - n\mu'_{11}\beta^0_{11}e^0_{15})\xi\zeta^2\tanh(\xi h)\Big)/ \\ & \Big(\kappa'_{11}\mu'_{11}\xi^2 + (\kappa'_{11}\mu^0_{11} + \kappa^0_{11}\mu'_{11})\xi\zeta\tanh(\xi h) + (\kappa^0_{11}\mu^0_{11} - (\beta^0_{11})^2)\zeta^2(\tanh(\xi h))^2\Big) \end{aligned} \tag{27}$$

Substituting Eq.(17) and Eq.(20) into Eq.(27) leads to the following wave velocity equation:

$$\frac{k + \sqrt{k^2 + 4(1 + \frac{\sigma^0_2}{\bar{c}^0_{44}})\left(1 - \frac{v^2}{v^2_A} + \frac{\sigma^0_1}{\bar{c}^0_{44}}\right)\xi^2}}{2\left(1 + \frac{\sigma^0_2}{\bar{c}^0_{44}}\right)} - \frac{\mu'\xi\sqrt{\frac{v^2}{v^2_B} - 1}\tan\left(\sqrt{\frac{v^2}{v^2_B} - 1}\xi h\right)}{\bar{c}^0_{44}} = \frac{M}{\bar{c}^0_{44}}, \tag{28}$$

where

$$\begin{aligned} M = \Big(& (m\kappa'_{11}\mu'_{11}e^0_{15} + n\kappa'_{11}\mu'_{11}h^0_{15})\xi^2\zeta \\ & + (m\kappa'_{11}\mu^0_{11}e^0_{15} + n\mu'_{11}\kappa^0_{11}h^0_{15} - m\kappa'_{11}\beta^0_{11}h^0_{15} - n\mu'_{11}\beta^0_{11}e^0_{15})\xi\zeta^2\tanh(\xi h)\Big)/ \\ & \Big(\kappa'_{11}\mu'_{11}\xi^2 + (\kappa'_{11}\mu^0_{11} + \kappa^0_{11}\mu'_{11})\xi\zeta\tanh(\xi h) + (\kappa^0_{11}\mu^0_{11} - (\beta^0_{11})^2)\zeta^2(\tanh(\xi h))^2\Big) \end{aligned}$$

The mechanical traction-free, magnetically and electrically open circuit conditions at $y = -h$ and the continuous conditions at $y = 0$ satisfy:

$$
(29)\quad
\begin{aligned}
&\sigma_B(x,-h,t) = 0, && \\
&\varphi_B(x,-h,t) = \varphi_C(x,-h,t), && D_B(x,-h,t) = D_C(x,-h,t), \\
&\phi_B(x,-h,t) = \phi_C(x,-h,t), && B_B(x,-h,t) = B_C(x,-h,t), \\
&\sigma_A(x,0,t) = \sigma_B(x,0,t), && w(x,0,t) = w'(x,0,t), \\
&\varphi_A(x,0,t) = \varphi_B(x,0,t), && D_A(x,0,t) = D_B(x,0,t), \\
&\phi_A(x,0,t) = \phi_B(x,0,t), && B_A(x,0,t) = B_B(x,0,t),
\end{aligned}
$$

which results in the algebraic equations in the unknowns A_1, A_2, A_3, B_1, B_2, B_3, B_4, B_5, B_6, C_1, C_2:

$$
(30)\quad
\begin{aligned}
&-e^{i\eta' h}B_1 + e^{-i\eta' h}B_2 = 0, \\
&e^{\xi h}B_3 + e^{-\xi h}B_4 = e^{-\xi h}C_1, \\
&\kappa'_{11}\xi e^{\xi h}B_3 - \kappa'_{11}\xi e^{-\xi h}B_4 = -\kappa_0\xi e^{-\xi h}C_1, \\
&e^{\xi h}B_5 + e^{-\xi h}B_6 = e^{-\xi h}C_2, \\
&\mu'_{11}\xi e^{\xi h}B_5 - \mu'_{11}\xi e^{-\xi h}B_6 = -\mu_0\xi e^{-\xi h}C_2, \\
&-(\bar{c}^0_{44}\eta A_1 + e^0_{15}\zeta A_2 + h^0_{15}\zeta A_3) = i\mu'\eta'(-B_1 + B_2), \\
&A_1 = B_1 + B_2, \\
&mA_1 + A_2 = B_3 + B_4, \\
&\kappa^0_{11}\zeta A_2 + \beta^0_{11}\zeta A_3 = \kappa'_{11}\xi B_3 - \kappa'_{11}\xi B_4, \\
&nA_1 + A_3 = B_5 + B_6, \\
&\beta^0_{11}\zeta A_2 + \mu^0_{11}\zeta A_3 = \mu'_{11}\xi B_5 - \mu'_{11}\xi B_6.
\end{aligned}
$$

The nontrivial solution exists only if the determinants of the coefficient matrix of Eq.(30) equals to zero, i.e.:

(31)

$$\begin{vmatrix} 0 & 0 & 0 & -e^{i\eta' h} & e^{-i\eta' h} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & e^{\xi h} & e^{-\xi h} & 0 & 0 & -e^{-\xi h} & 0 \\ 0 & 0 & 0 & 0 & 0 & \kappa'_{11}\xi e^{\xi h} & -\kappa'_{11}\xi e^{-\xi h} & 0 & 0 & \kappa_0\xi e^{-\xi h} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & e^{\xi h} & e^{-\xi h} & 0 & -e^{-\xi h} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mu'_{11}\xi e^{\xi h} & -\mu'_{11}\xi e^{-\xi h} & 0 & \mu_0\xi e^{-\xi h} \\ -\bar{c}^0_{44}\eta & -e^0_{15}\zeta & -h^0_{15}\zeta & i\mu'\eta' & -i\mu'\eta' & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ m & 1 & 0 & 0 & 0 & -1 & -1 & 0 & 0 & 0 & 0 \\ 0 & \kappa^0_{11}\zeta & \beta^0_{11}\zeta & 0 & 0 & -\kappa'_{11}\xi & \kappa'_{11}\xi & 0 & 0 & 0 & 0 \\ n & 0 & 1 & 0 & 0 & 0 & 0 & -1 & -1 & 0 & 0 \\ 0 & \beta^0_{11}\zeta & \mu^0_{11}\zeta & 0 & 0 & 0 & 0 & -\mu'_{11}\xi & \mu'_{11}\xi & 0 & 0 \end{vmatrix} = 0$$

From Eq.(31), we have:

(32)
$$\bar{c}^0_{44}\eta - \mu'\eta'\tan(\eta' h) = \Big(nb\mu'_{11}\xi\zeta(\kappa^0_{11}h^0_{15} - e^0_{15}\beta^0_{11}) + ma\kappa'_{11}\xi\zeta(e^0_{15}\mu^0_{11} - h^0_{15}\beta^0_{11}) + \kappa'_{11}\mu'_{11}\xi^2 ab(nh^0_{15} + me^0_{15})\Big) \Big/ \Big(\zeta(\kappa^0_{11}\mu^0_{11} - (\beta^0_{11})^2) + \xi(a\mu^0_{11}\kappa'_{11} + b\kappa^0_{11}\mu'_{11})\Big),$$

where: $a = \dfrac{\kappa'_{11}\tanh(\xi h) + \kappa_0}{\kappa'_{11} + \kappa_0\tanh(\xi h)}$ and $b = \dfrac{\mu'_{11}\tanh(\xi h) + \mu_0}{\mu'_{11} + \mu_0\tanh(\xi h)}$. Substituting Eq.(17) and Eq.(20) into Eq.(32) leads to the following wave velocity equation:

(33)
$$\frac{k + \sqrt{k^2 + 4\left(1 + \frac{\sigma^0_2}{\bar{c}^0_{44}}\right)\left(1 - \frac{v^2}{v_A^2} + \frac{\sigma^0_1}{\bar{c}^0_{44}}\right)\xi^2}}{2\left(1 + \frac{\sigma^0_2}{\bar{c}^0_{44}}\right)} - \frac{\mu'\xi\sqrt{\frac{v^2}{v_B^2} - 1}\tan\left(\sqrt{\frac{v^2}{v_B^2} - 1}\,\xi h\right)}{\bar{c}^0_{44}} = \frac{N}{\bar{c}^0_{44}},$$

where:

$$N = \Big(nb\mu'_{11}\xi\zeta(\kappa^0_{11}h^0_{15} - e^0_{15}\beta^0_{11}) + ma\kappa'_{11}\xi\zeta(e^0_{15}\mu^0_{11} - h^0_{15}\beta^0_{11}) + \kappa'_{11}\mu'_{11}\xi^2 ab(nh^0_{15} + me^0_{15})\Big) \Big/ \Big(\zeta(\kappa^0_{11}\mu^0_{11} - (\beta^0_{11})^2) + \xi(a\mu^0_{11}\kappa'_{11} + b\kappa^0_{11}\mu'_{11})\Big).$$

4. Numerical results and discussions

In the following numerical examples, the thickness of the homogeneous layer h is $1\times10^{-2}m$. Consider the elastic material Si in the homogeneous layer and the piezomagnetic material $CoFe_2O_4$ in the functionally graded substrate. The material constants are listed in Table 1. In the numerical examples, the wave speed of Love wave at different gradient index and different initial stresses are computed and the results are shown graphically. The influence of the gradient index and the initial stress are discussed based on the numerical results.

Table 1. Material parameters used in the computation [20–21]

Materials	c_{44}/ $(10^9\text{N}\cdot\text{m}^{-2})$	ρ/ $(10^3\text{kg}\cdot\text{m}^{-3})$	κ_{11}/ $(10^{-9}\text{C}^2\cdot\text{N}^{-1}\cdot\text{m}^{-1})$	μ_{11}/ $(10^{-6}\text{Ns}^2\cdot\text{C}^{-2})$	h_{15}/ $(\text{N}\cdot\text{A}^{-1}\cdot\text{m}^{-1})$
Elastic layer (Si)	79.4	2.328	0.1035	1.256	0
Functionally graded substrate ($CoFe_2O_4$)	45.3	5.3	0.08	157	550

First, the surface wave speed at different gradient index and the fixed initial stresses σ_2^0 is computed and the results are shown in Fig. 2. It is found that the surface wave speed is very sensitive to the gradient index. Furthermore, whether the boundary condition is a short circuit or open circuit, the

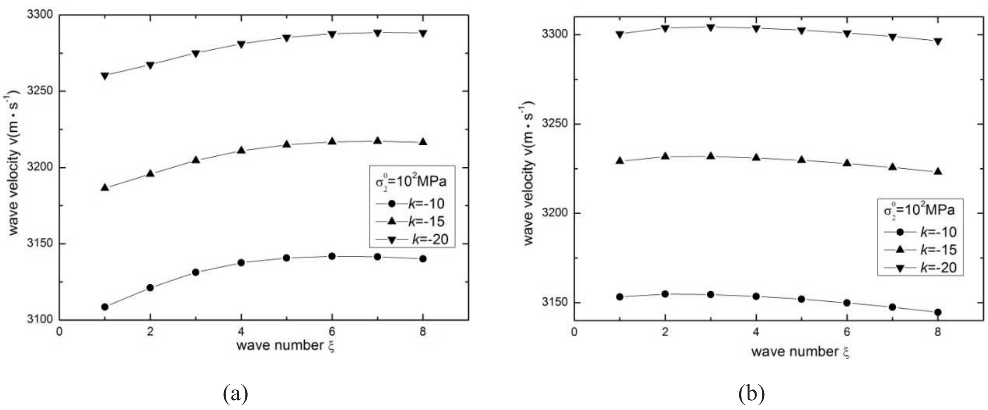


Fig. 2. The surface wave velocities of the piezomagnetic material ($CoFe_2O_4$) with the fixed initial stress σ_2^0: (a) short circuit condition; (b) open circuit condition

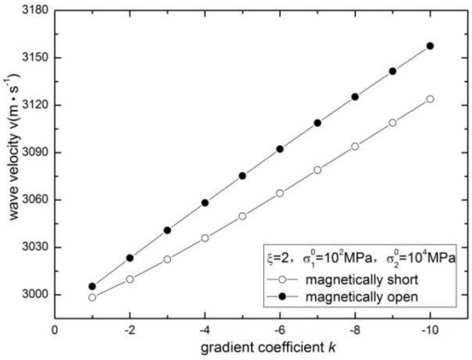


Fig. 3. The surface wave velocities of the piezomagnetic material ($CoFe_2O_4$) at different gradient index and magnetically surface boundary

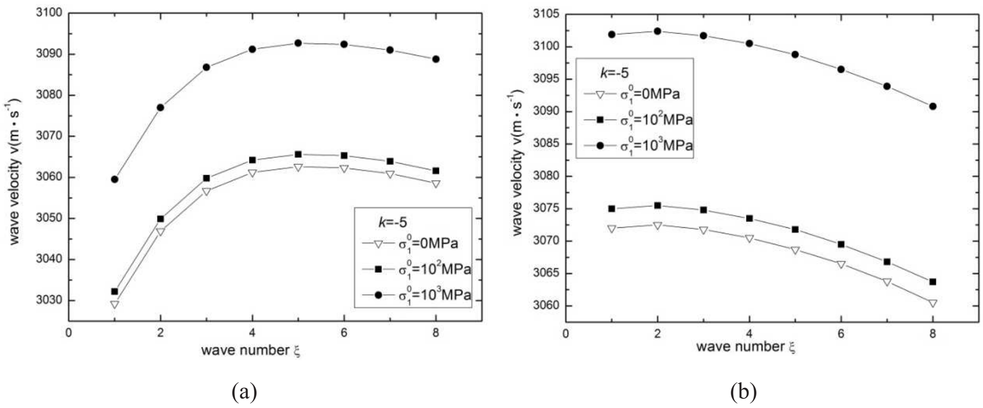
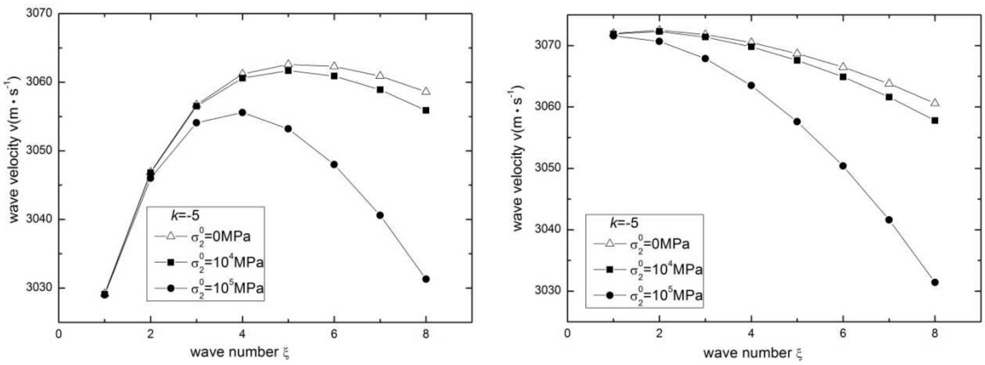


Fig. 4. The surface wave velocities of the piezomagnetic material ($CoFe_2O_4$) with the fixed gradient index: (a) short circuit condition, (b) open circuit condition

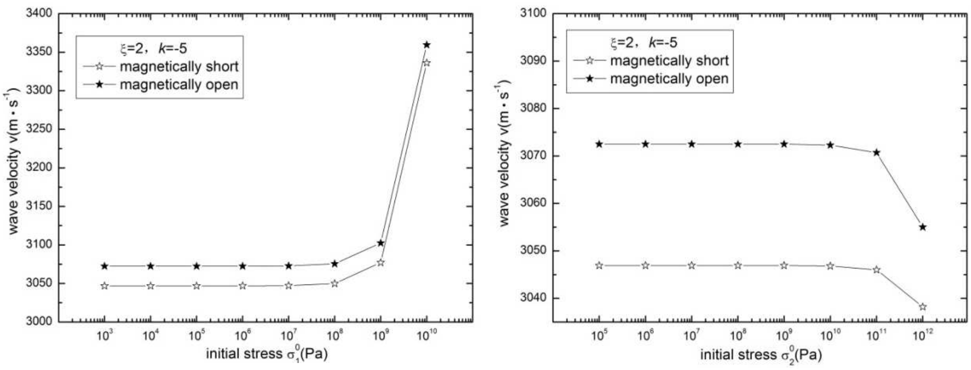
surface wave speed increases gradually with the increase of the absolute value of gradient index for the piezomagnetic medium. Similar computations are performed for the fixed initial stresses σ_1^0 and similar trend is observed. It can be seen, that the surface wave speed is more sensitive under magnetically open circuit condition than under magnetically short circuit condition and the surface wave speed under magnetically open circuit condition is a little bit higher than under magnetically short circuit condition at the same frequency. This is shown in Fig. 3.

The influences of the initial stresses on the surface wave speed under the fixed gradient index are shown in Figs 4, 5 and 6. It is found, that the initial



(a) (b)

Fig. 5. The surface wave velocities of the piezomagnetic material ($CoFe_2O_4$) with the fixed gradient index: (a) short circuit condition, (b) open circuit condition



(a) (b)

Fig. 6. The surface wave velocities of the piezomagnetic material ($CoFe_2O_4$) at different initial stress and magnetically surface boundary: (a) at different initial stressσ_1^0; (b) at different initial stressσ_2^0

stress σ_1^0 makes the surface wave speed increasing, while the initial stress σ_2^0 makes the surface wave speed decreasing. Furthermore, the effect of initial stress σ_1^0 is the same at different frequency, and the effect of initial stress σ_2^0 is more evident at high frequency, than at low frequency. Namely, the effect of initial stress σ_1^0 is frequency-independent, while the effect of initial stress σ_2^0 is frequency-dependent. Although the existence of initial stress σ_1^0 and σ_2^0 can make the surface wave speed changed, but this change is evident only when the value of σ_1^0 and σ_2^0 approach the value of c_{44}. This trend is shown in Fig.

6. The comparison of the effect of σ_1^0andσ_2^0shows that the effect of $\sigma_1^0$is more evident than σ_2^0. Therefore, if the initial stress is used to enhance the surface wave speed, the imposing σ_1^0 along the direction parallel to the free surface is better than the imposing σ_2^0 along the direction normal to the free surface.

4. Conclusion

The Love wave can exist at the homogeneous layered half-infinite magnetic-electric-elastic medium. Whether the boundary condition is short circuit or open circuit, the surface wave speed increases gradually with the increase of the absolute value of gradient index. But the surface wave speed is more sensitive to the gradient index under the open circuit condition than under the short circuit condition. The initial stress has evident influence on the surface wave speed. In general, the existence of the initial stress parallel to the surface has more evident influence than the initial stress perpendicular to the surface. Furthermore, the existence of the initial stress parallel to the surface makes the surface wave speed increasing but the existence of the initial stress perpendicular to the surface makes the surface wave speed decreasing. However, only when the initial stress approaches to the magnitude of elastic constants the effects of initial stress on the surface wave speed are pronounced.

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