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DIAGNOSIS OF TYPE, LOCATION AND SIZE OF CRACKS BY USING GENERALIZED DIFFERENTIAL QUADRATURE AND RAYLEIGH QUOTIENT METHODS*

Majid Akbarzadeh Khorshidi
School of Mechanical Engineering, College of Engineering, University of Tehran, Iran,
e-mail: majid.akbarzadeh.kh@gmail.com

Delara Soltani
School of Mechanical Engineering, Mazandaran Institute of Technology, Iran,
e-mail: delara.soltani@gmail.com

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Abstract. In this paper, an appropriate and accurate algorithm is proposed to diagnosis of lateral or vertical cracks on beam, based on beam natural frequencies. Clamped-free boundary conditions are assumed for the beam. The crack in beam is modelled by without mass torsion spring. Then, the relationship between the beam natural frequencies, location and stiffness of the crack is presented by using the Rayleigh quotient and the governing equation is solved by using generalized differential quadrature method (GDQM). If there is only one crack in the beam, then three natural frequencies are used as inputs to the algorithm and mode shapes corresponding to each the natural frequencies are calculated. Finally, type, location and severity of cracks in beam, are diagnosed.
Key words: Crack detection, cantilever beam, natural frequency, GDQ method, Rayleigh quotient.

1. Introduction

One of the main causes of failure of structures is the existence of cracks in them. Cracks should be diagnosed when they have little depth in order to prevent crack growth and sudden failure of structures. Although, several methods such as ultrasonic, acoustic emission, X-ray, etc., can be used for crack detection, but using these methods for large and complex structures are

*Corresponding author e-mail: majid.akbarzadeh.kh@gmail.com

difficult [1, 2]. Therefore, crack diagnosis techniques by vibration have found widespread application [3].

Nomenclature

a Crack depth
B width of beam
E Modulus of elasticity
H Height of beam
I Moment of inertia of beam
k Torsional spring stiffness
L Length of beam
m Bending moment
n Beam vibration mode number
u_n Beam strain energy

Greek symbol

$\beta = x/L$ Dimensionless location of crack
$\alpha = a/H$ Dimensionless depth of crack
ω_n Natural frequency
$\phi_n(\beta)$ Beam mode shape in nth mode

Beams are common structures used to carry and transfer high loads in industry. Sudden failure during high load operation may cause serious damage or injury, so early crack detection is important. Vibration measurements offer a non-destructive, inexpensive and fast means to detect and locate cracks, by Lee and Chung [4]. The presence of a crack or localized damage in a structure reduces the stiffness and increases the damping in the structure. Vibration theory states, reduction in the stiffness is associated with decrease in the natural frequencies and the modes modification of the structure vibration [5].

Many useful research and activities are performed in the field of crack detection. Detection of the location and size of cracks in the multiple cracked beam by spatial wavelet based approach, is performed by Chang and Chen [6]. In the discussion of crack detection, many activities are presented using finite element methods [7]. Ma et al. [8] have constructed one-dimensional Daubechies wavelet-based beam elements for the beam bending problems. Chen et al. [9] have presented two-dimensional wavelet finite elements based on Daubechies wavelets and applied them on the plane-stress and plane-strain problems.

In this paper, an algorithm for the recognition of lateral or vertical cracks on cantilever beam is proposed. In fact, this method is very simple, because from vibrations approach for crack detection are used and the method is also very exact, because an accurate numerical approach for solving the equations is employed. In this presentation, GDQ method has been used to solve the equations. GDQ method is one of the most useful methods for numerical solving the differential equations of solid mechanics [10]. By using the Rayleigh quotient, the relationship between beam natural frequencies, location and stiffness of the crack is presented and location and severity of cracks are detected. Finally, type of cracks in beam, are diagnosed by the difference between the hardness of transverse and vertical cracks.

2. Model

According to Fig. 1, if the transverse vibrations of beam in direction 1 be considered, crack No. 1, transverse crack and crack No. 2 the vertical crack is called. The crack in the beam can be modelled by the torsion spring. In this case, reducing the beam strain energy due to the crack, with the following equation can be expressed:

(1)
$$\Delta u_n = \frac{m_n^2}{2k},$$

where m_n is bending moment for the crack location, n is mode number and k is equivalent torsion spring stiffness of the crack.

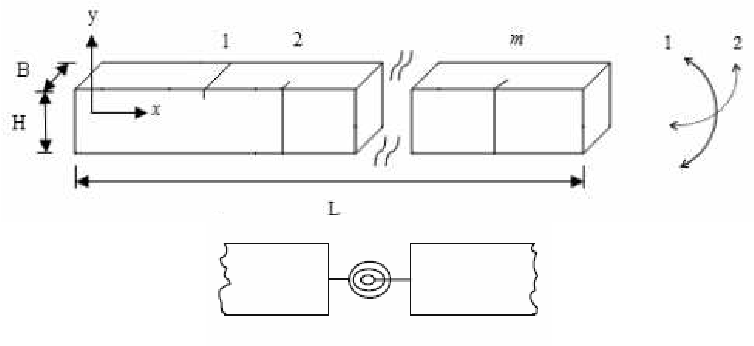

Fig. 1. Display two types of cracks in beam and vibration directions

The equivalent stiffness of cracks are demonstrated for rectangular cross section of beams under bending by using the stress intensity factor and the energy consumed. It is also assumed, that the cracks occur in the first mode of fracture (the opening mode of fracture). For the transverse crack, equivalent stiffness with the following equation is expressed by Ostachowicz et al. [11]:

$$k_t = \frac{EBH^2}{72\pi F(\alpha_1)}. \tag{2}$$

$$F(\alpha_1) = 0.6384\alpha_t^2 - 1.035\alpha_t^3 + 3.7201\alpha_t^4 - 5.1774\alpha_t^5 - 7.332\alpha_t^7 + 2.4909\alpha_t^8. \tag{3}$$

$$\alpha_t = fraca_t H, \tag{4}$$

where F is corrective function and α_t is the dimensionless depth of the transverse crack. The equivalent spring constant for the vertical crack can be written as [12]:

$$k_v = \frac{EH^3}{9\pi F(1.12)^2\alpha_v^2}, \tag{5}$$

$$\alpha_v = \frac{a_v}{B}. \tag{6}$$

Therefore, we have:

$$\Delta u_n = \frac{m_n^2}{2k_t}, \tag{7}$$

$$\Delta u_n = \frac{m_n^2}{2k_v}. \tag{8}$$

The Rayleigh quotient is used for relationship between beam natural frequencies and crack properties, that for beam with a crack the following equation obtained by Patil et al. [13]:

$$\frac{\Delta\omega_n}{\omega_n} \cong \frac{\left(\frac{m_n^2}{2k}\right)}{2u_n}, \tag{9}$$

where ω_n is natural frequency of normal beam and $\Delta\omega_n$ is the difference between natural frequencies of normal and cracked beam.

Strain energy u_n of the normal beam and β dimensionless location of crack, are obtained from the following equation:

$$u_n = \frac{L}{2}\int_0^1 EI[\phi_n''(\beta)]^2 d\beta, \tag{10}$$

$$\beta = \frac{x}{L}, \tag{11}$$

and

$$m_n = EI\phi_n''^2(\beta), \tag{12}$$

where ϕ_n is beam mode shape for normal beam.

3. Calculation

With inserting Eqs. (10) and (12) in Eq. (9), the following differential equation is formed:

$$\frac{\Delta\omega_n}{\omega_n} = \frac{EI\phi_n''^2(\beta)}{2kL\int_0^1 [\phi_n''^2(\beta)]d\beta}. \tag{13}$$

Two natural frequencies, mode shapes corresponding to each of the natural frequencies are calculated using the GDQ method. With this solution, crack depth and location is determined but crack is not clear whether it is a transverse or a vertical one, to solve this problem, we use the third natural frequency that is obtained from direction 2 of the beam. With using this frequency, we have third equation:

$$\frac{\Delta\omega_{2,1}}{\omega_{2,1}} = \frac{EI\phi_{2,1}''^2(\beta)}{2k_2L\int_0^1 [\phi_{2,1}''^2(\beta)]d\beta}, \tag{14}$$

where k_2 is spring stiffness in direction 2, also in $\omega_{i,j}$, i and j are vibration mode and vibration direction, respectively.

From Eq. (14), value of x is obtained. We can recognize the type of crack by using k_1 and k_2. Prove that for a certain range of values B and H, by changing the beam vibration direction, transverse crack stiffness is always less than the vertical crack stiffness. This range is as follows:

$$0.498 < \frac{B}{H} < 1.87. \tag{15}$$

So, if $k_1 < k_2$ obtained, then crack in direction 1 is the transverse one. Crack type is detectable by using the above results.

4. Example and results

In this section, a beam with the following specifications is used:

$$\begin{aligned} E &= 210 \text{ GPa}; \\ v &= 0.3; \\ B &= 12.5 \text{ mm}; \\ \rho &= 7860 \text{ kg/m}^3; \\ l &= 150 \text{ mm}; \\ H &= 25 \text{ mm}. \end{aligned}$$

Natural frequencies for normal beam:

$$\begin{aligned} \omega_{1,1} &= 868.76 \text{ Hz}; \\ \omega_{1,2} &= 12502.34 \text{ Hz}; \\ \omega_{2,1} &= 4342.24 \text{ Hz}. \end{aligned}$$

For example, consider the case of transverse crack. The results of solving equations come in the form below:

$$\beta = 0.4986, \quad k_1 = 0.9604 \times 10^5.$$

Now, by third frequency, k_2 is obtained:

$$k_2 = 1.3011 \times 10^5.$$

Since $k_2 > k_1$, therefore crack is in direction 1. Using Eq. (2) crack depth is calculated:

$$\alpha_t = 0.3526.$$

Natural frequency for beam with transverse crack:

$$\begin{aligned} \omega_{1,1} &= 865.37 \text{ Hz}; \\ \omega_{1,2} &= 12492.3 \text{ Hz}; \\ \omega_{2,1} &= 4321.77 \text{ Hz}. \end{aligned}$$

The same process is performed for the vertical crack. Natural frequency for beam with vertical crack:

$$\begin{aligned} \beta &= 0.0950, \\ \alpha &= 0.1905, \\ \omega_{1,1} &= 861.70 \text{ Hz}, \\ \omega_{1,2} &= 12486.04 \text{ Hz}; \\ \omega_{2,1} &= 4316.03 \text{ Hz}. \end{aligned}$$

The results of the presented algorithm are compared with the obtained results in available literature [13]. From comparison of results, we recognize that the results of present algorithm are close to the experimentally results. Table 1 reports a comparison of results for α and β for several input natural frequencies. The accuracy of the presented algorithm is shown in Fig. 2.

Table 1. Results of presented algorithm

Normal beam			*Cracked beam*				
$\omega_{1,1}$	$\omega_{1,2}$	$\omega_{2,1}$	$\omega_{1,1}$	$\omega_{1,2}$	$\omega_{2,1}$	α	β
625.32	11899.04	5442.12	611.58	11842.42	5033.12	0.498	0.193
759.40	11611.01	4052.32	742.86	11983.91	4048.12	0.492	0.409
912.24	12590.12	4890.56	896.03	12545.01	4810.61	0.210	0.510
868.76	12502.34	4342.24	865.37	12492.3	4321.77	0.352	0.498
903.55	10215.76	3960.32	890.36	10195.34	3946.15	0.502	0.701
822.09	12385.93	3692.87	806.02	12376.06	3682.01	0.495	0.497

5. Conclusion

In this paper, a new algorithm is presented for detecting the type of transverse or vertical crack in beam and predicting the location and size of cracks. The algorithm due to use of GDQM, is calculated with high accuracy. Results are compared with obtained results by other authors in available literature. The obtained results show that the proposed algorithm has good accuracy, because error is small in comparison with experimental results. This algorithm is easily generalized for the beam with multiple cracks.

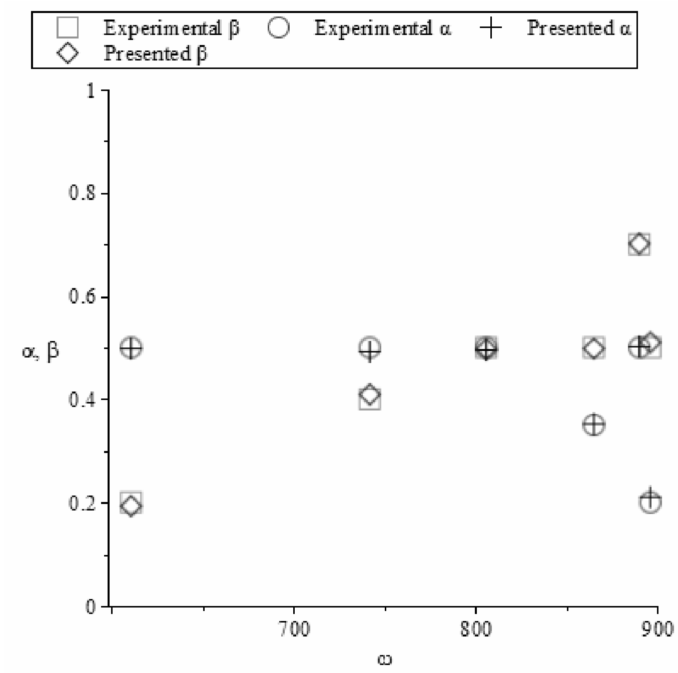

Fig. 2. Comparison of presented results with experimental results

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Appendix

Calculate equivalent torsion spring stiffness:

Reducing of stored elastic energy in beam due to crack under plane stress is shown as follows [15]:

$$\Delta u = \frac{1}{E}\int_0^1 K_I^2 dA, \tag{A1}$$

where K_I is stress intensity factor, E is modulus of elasticity and A is crack surface.

Also, stress intensity factor calculate as follows:

$$K_I = \sigma\sqrt{\pi a} g\left(\frac{a}{H}\right), \tag{A2}$$

where σ is stress in crack zone, a is crack depth, H is beam height and $g\left(\frac{a}{H}\right)$ is a correction function, that is proposed by Anifantis et al. [16].

$$g\left(\frac{a}{H}\right) = 1.13 - 1.374\left(\frac{a}{H}\right) + 5.749\left(\frac{a}{H}\right)^2 - 4.464\left(\frac{a}{H}\right)^3. \tag{A3}$$

By use of mechanical relations $\left(\Delta u = \frac{m^2}{2K},\ \sigma = \frac{6m}{BH^2} \text{ and } dA = Bda\right)$, Eq. $(A4)$ is obtained.

$$k = \frac{EBH^2}{72\pi f\left(\frac{a}{H}\right)}. \tag{A4}$$

In Eq. (A4) k is spring stiffness and $f\left(\frac{a}{H}\right)$ is as follows:

$$f\left(\frac{a}{H}\right) = 0.6384\left(\frac{a}{H}\right)^2 - 1.035\left(\frac{a}{H}\right)^3 + 3.7201\left(\frac{a}{H}\right)^4 - \cdots \tag{A5}$$