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# ANALYTICAL, NUMERICAL AND EXPERIMENTAL EXAMINATION OF REINFORCED COMPOSITES BEAMS COVERED WITH CARBON FIBER REINFORCED PLASTIC

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Abstract. In the article, analytical, numerical (Finite Element Method) and experimental investigation results of beam that was strengthened with fiber reinforced plastic-FRP composite has been given as comparative, the effect of FRP wrapping number to the maximum load and moment capacity has been evaluated depending on this results. Carbon FRP qualitative dependences have been occurred between wrapping number and beam load and moment capacity for repair-strengthen the reinforced concrete beams with carbon fiber. Shown possibilities of application traditional known analysis programs, for the analysis of Carbon Fiber Reinforced Plastic (CFRP) strengthened structures.
Key words: Carbon Fiber Reinforced Plastic (CFRP) strengthened structures, finite element method, moment capacity, maximum load.

## 1. Introduction

The existing buildings that are under earthquake action, partly beams are suffering damage. On the other hand, while determining the performance of the existing building during seismic formation that can occur, as for as possible without increasing the mass of the building, the imperativeness of strengthening the beams comes out [5]. At each condition, consequently at either repair on strengthen studies, determining the connection between technical repair procedures and the beam capacity comes into prominence. In this direction, studies are being done by means of being looked from different point of view a direction [12].

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In recent years, for the aim of repairing and strengthening, applications of fiber reinforced plastic composite system by gluing them to external part of the reinforced concrete structures gradually becomes widespread all over the world [15, 6]. Fibers that have most prevalent usage are e-glass, aramid and carbon. This are manufactured as plaques that were covered with fibers and as tissues that were knitted in one and two directions. Behaviour of the system that was covered external with FRP composite is related to the type of the element that was covered and generally that have been separated into three categories that are bending strengthening, shear strengthening and envelope scripts [14, 4].

Strengthening reinforced concrete structures includes external covering of beams towards bending and shearing and external seismic covering of columns and beams. The aim of seismic external covering increases the resistance and the ductility of the system towards lateral earthquake loads [10].

Behaviour of the reinforced concrete beam with "T" cross section that was strengthened with carbon fiber reinforced plastic composite (CFRP) has been evaluated analytical and experimental, it has been observed that tension increased approximately %40 in [11] study, at the negative moment region.

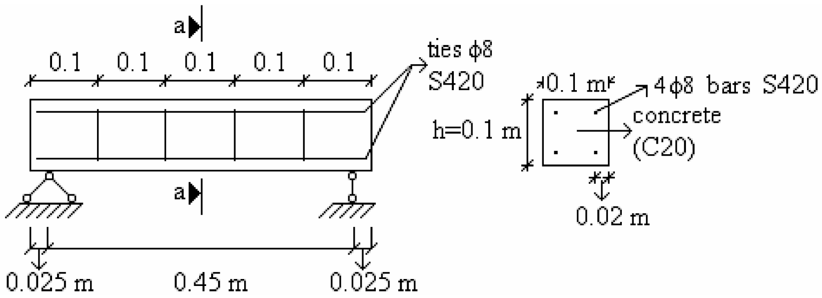
Distance from support to CFRP origin and effect of cross-section beam to behaviour of it have been observed in [2] study, at the tensile region of reinforced concrete beam when it was strengthened with CFRP composite. Computation formula has been composed related to experiment results, to guess the design load that is equal to the limit position of beam. In this examination original shear stress and slight effect have been taken into consideration.

Behaviour of partial bridges that was strengthened with CFRP composite has been examined in [13] study. On scaled specimen and full-scaled partial beams experiments were done. Bond scaled experiment have been shown as alternative for characterizing repair and strengthen the partial structures with CFRP composite.

Experimental results of repair-strengthen with CFRP composite have been presented in [8] study, at the example of pre-stressed three reinforced concrete girder bridge that suffered damage. Before and after repair experiment results have shown that usage of CFRP is productive. It has been observed that usage of CFRP decreased the girder bending displacement more than %20.

## 2. Specimen details

Specimen reinforced concrete girder and bars in it and measurements of ties have been shown in Fig. 1. $\phi 8$ longitudinal bar and $\phi 8$ bar as tie have been used in girder.




(a) beam (b) a-a cross section

Fig. 1. Measurement of specimen and details of bars

In Table 1 a, b physical parameters of concrete, bar and CF-130 type of CFRP: Concrete compressive strength – $f_c$, approximate elastic modulus of concrete in compression – $E_c$, Poisson' s ratio – $\mu_c$; design strength of the FRP material – $f_{fu}$, tension (fracture) extension – $\varepsilon_{su}$, modulus of elasticity – $E_{st}$, Poisson' s ratio – $\mu_{st}$, thickness of CFRP – $t_f$, modulus of Elasticity – $E_f$, ultimate strain (elongation) of the FRP material – $\varepsilon_{fu}$ have been given.

Table 1a. Characteristics of concrete and bar

| Concrete (C20) | | | Bars (S420) | | | |
|---|---|---|---|---|---|---|
| $f_c$ (MPa) | $E_c$ (MPa) | $\mu_c$ | $f_y$ (MPa) | $\varepsilon_{su}$ | $E_{st}$ (MPa) | $\mu_{st}$ |
| 20 | $28 \cdot 10^3$ | 0.2 | 420 | 0.003 | $2 \cdot 10^5$ | 0.3 |

Table 1b. Characteristics of CF-130 type of CFRP

| $t_f$ (m) | $f_{fu}$ (Mpa) | $\varepsilon^*_{fu}$ | $E_f$ (Mpa) |
|---|---|---|---|
| $1.65 \cdot 10^{-4}$ | 3430 | 0.015 | $2.3 \cdot 10^5$ |

Following direction observed for covering all specimen reinforced concrete beams with CFRP (CF-130), [1].

1. Preparation of surface: After cleaning the surface, it has been prepared for placing by means of sand blasting.

2. Application of primer: One layer primer has been applied to concrete surface. Preparation of primer on concrete surface is for application of CFRP layers.

3. Application of putty: A thin coat of putty is smoothed over the

surface to fill in any small uneven surfaces.

4. Application of saturant: This first layer is applied before placing of CFRP.

5. Application of CFRP layers: First CFRP layer is placed and layers are rolled into the saturant to insure good adhesion.

6. Application of the second layer: For the second placing it is necessary to be impregnated for saturant between layers.

Not to cut FRP, corners of specimen have been become circular.

## 3. Experiment plan

3 of experiment specimens (RB01, RB02, RB03) without CFRP, 3 of them (RB11, RB12, RB13) with one layer, 3 of them (RB21, RB22, RB23) with two layers, 3 of them (RB31, RB32, RB33) with three layers, 3 of them (RB41, RB42, RB43) with four layers CFRP have been prepared by gluing them to tension region. Measurement instruments of deformations and displacements have been placed at tension region in the (PFL-90-11) of specimen (Fig. 2.)

## 4. Test result

In Table 2, the results of load capacity ($P$), moment capacity ($M$), maximum deformation ($\varepsilon_c, \varepsilon_f$) and displacement ($u$) of concrete and fiber in the middle of specimen, curvature (appropriate to this position) for the failure mode have been given.

Table 2

| Specimen name | $P$ (kN) | $M$ (kNm) | $\varepsilon_c/10^{-3}$ | $\varepsilon_f/10^{-3}$ | $u$ (mm) | $\phi/10^{-3}$ (Rad/m) | Fracture mode |
|---|---|---|---|---|---|---|---|
| RB0 | 30.54 | 2.29 | 0.22 | 0.42 | 2.92 | 6.875 | 1 |
| RB1 | 46.30 | 3.47 | 0.49 | 0.56 | 1.69 | 14.41 | 2 |
| RB2 | 53.69 | 4.02 | 0.47 | 0.24 | 1.71 | 11.46 | 2 |
| RB3 | 60.09 | 4.50 | 0.41 | 0.12 | 1.00 | 9.11 | 3 |
| RB4 | 64.03 | 4.80 | 0.30 | 0.14 | 0.83 | 6.12 | 3 |

In Table 2 legend of indication 1, 2, 3 appropriately were given below:

1 – Fracture mode that was occurred after yielding of tension bar.

2 – Fracture mode that was occurred after peeling of CFRP layer.

3 – Fracture mode that was occurred after peeling of CFRP layer and crushing of concrete.

In Table 2, beam bearing moment ($M$) related to beam bearing load ($P$), has been found from following equations for plastic failure situation:

$$\operatorname{tg}\theta \approx \theta = u_p/\left(L/3\right); \quad u_p = \theta \cdot L/3$$

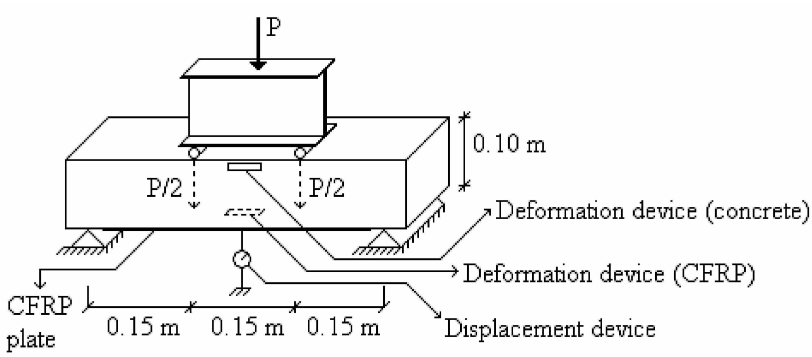



Fig. 2a. Placement positions of deformation and displacement instruments on specimen

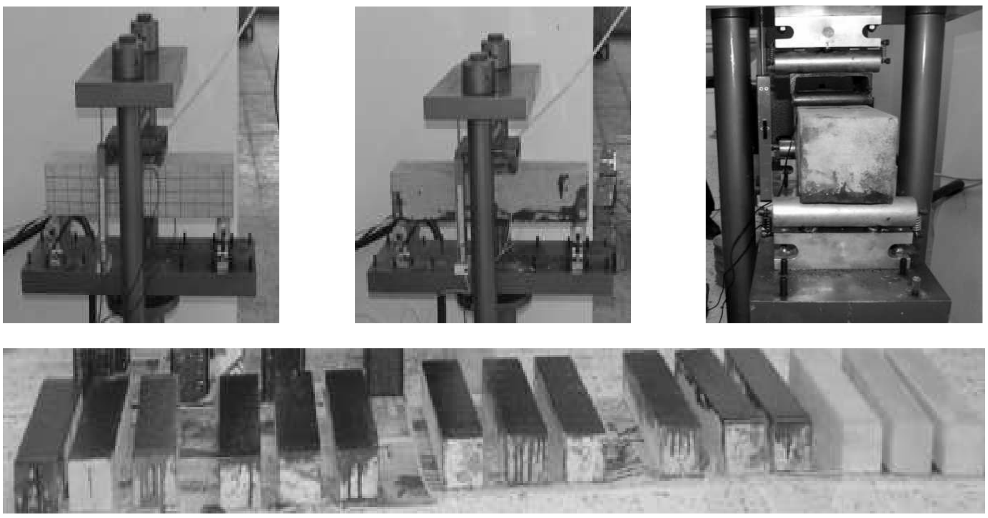


Fig. 2b. Specimens without and with CFRP during of experiment

$$2\frac{P}{2}\cdot\frac{L}{3}\cdot\theta = 2M\theta + 2M\cdot 0; \tag{1}$$

$$M = \frac{PL}{6} \quad \text{or} \quad P = \frac{6M}{L}. \tag{2}$$

Computation of beam curvature have been found from neutral axis depth ($c$) related to source of [3] and from concrete deformation that was

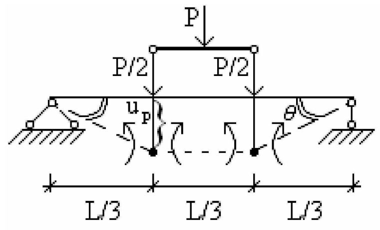



Fig. 3. Schematic projection of plastic failure of beam

found from experiment.

(3) $$\phi = \frac{\varepsilon_c}{c}.$$

Neutral axis depth values have been given in Table 3.

## 5. Analytical analysis

As it known bearing capacity of reinforced concrete beam with rectangular cross-section has been calculated as follows:

(4) $$M_u = m_u b d^2 f_{cd}.$$

Value of $m_u$:

(5) $$\rho_m = \rho \frac{f_{yd}}{f_{yk}} = \frac{A_s}{bd} \cdot \frac{f_{yd}}{f_{yk}}.$$

For testing specimens

(6) $$\rho_m = \frac{0.1 \cdot 0.1}{10 \cdot 0.8} = \frac{420000/1.15}{20000/1.5} = 0.013 \cdot 27.37 = 0.355,$$

for $\rho_m = 0.355$, from $m_u - \rho_m$ relation:

(7) $$m_u = 0.27.$$

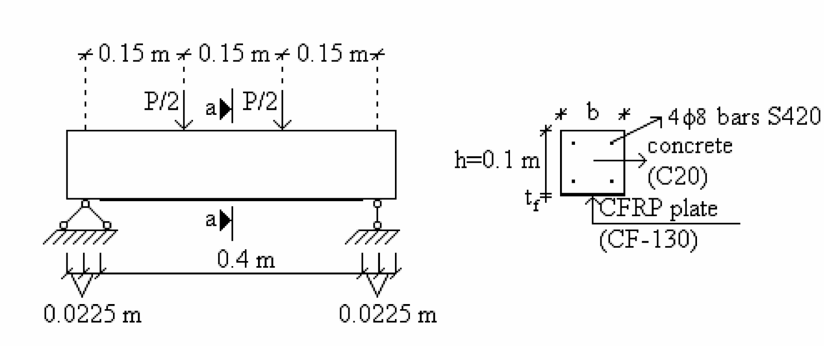
After finding $m_u$, it is written in formula (4) and bearing capacity of specimen is calculated:

(8) $$M_u = 0.27 \cdot 0.1 \cdot 0.08^2 \cdot 13333 = 2.3 \ \text{kN.m}.$$

Related to this bearing capacity, beam bearing load is found from formula (2).

$$P_u = \frac{6M_u}{L} = \frac{6 \cdot 2.3}{0.45} = 30.67 \text{ kN}. \tag{9}$$

Bearing capacity of reinforced concrete beam that was covered with CFRP on its tension (Fig. 4) has been computed as following related to source [3, 16].




(a) beam (b) a-a cross section

Fig. 4. Reinforced concrete beam with CFRP and its details of cross section

Properties of concrete, bar and fiber material is computed:
for fiber:

$$f_{fu} = C_E f_{fu}, \tag{10}$$

$$\varepsilon_{fu} = C_E \varepsilon_f, \tag{11}$$

$$A_f = n t_f w_f, \tag{12}$$

$$\rho_f = \frac{A_f}{bd}, \tag{13}$$

$$n_f = \frac{E_f}{E_c}, \tag{14}$$

for bar:

(15) $$\rho_s = \frac{A_s}{bd},$$

(16) $$n_s = \frac{E_s}{E_c}.$$

Position of neutral axis ($c$) has been obtained as follows in first approach:

(17) $$c = 0.2d.$$

Bending moment bearing capacity of beam cross-section has been found from following computation algorithm related to the preceding information's.

Calculation order of bearing capacity of beam ($M$) and $B_1$, $B_2$ algorithms and the value of $c_*$ have been shown as follows:

— Calculation of the $c_*$ parameter by $B_1$ algorithm:

(Crushing position of concrete)

(18) $$\varepsilon_c = \varepsilon_{cu},$$

(19) $$\varepsilon_s = \varepsilon_{cu}\left(\frac{d-c}{c}\right),$$

(20) $$\varepsilon'_s = \varepsilon_{cu}\left(\frac{c-d'}{c}\right),$$

(21) $$\varepsilon_f = \varepsilon_{cu}\left(\frac{h-c}{c}\right) - \varepsilon_{bi},$$

(22) $$f_s = E_s \varepsilon_\varepsilon \le f_y,$$

(23) $$f'_s = E_s \varepsilon'_s \le f_y,$$

(24) $$c_* = \frac{A_s f_s + A_f f_f - A'_s f'_s}{0.85 \beta_1 f'_c b}.$$

— Calculation of the $c_*$ parameter by $B_2$ algorithm:
(Failure by FRP ruptures)

$$\varepsilon_f = \varepsilon_{fu} = \varepsilon_b - \varepsilon_{bi}, \tag{25}$$

$$\varepsilon_c = (\varepsilon_{fu} + \varepsilon_{bi})\left(\frac{c}{h-c}\right), \tag{26}$$

$$\varepsilon_s = (\varepsilon_{fu} + \varepsilon_{bi})\left(\frac{d-c}{h-c}\right), \tag{27}$$

$$\varepsilon'_s = (\varepsilon_{fu} + \varepsilon_{bi})\left(\frac{c-d'}{h-c}\right), \tag{28}$$

$$\varepsilon'_c = \frac{1.71 f'_c}{E_c}, \tag{29}$$

$$\beta_1 = 2 - \frac{4\left[\frac{\varepsilon_c}{\varepsilon'_c} - \tan^{-1}\left(\frac{\varepsilon_c}{\varepsilon'_c}\right)\right]}{\left(\frac{\varepsilon_c}{\varepsilon'_c}\right)\ln\left(1 + \frac{\varepsilon_c^2}{(\varepsilon'_c)^2}\right)}, \tag{30}$$

$$\gamma = \frac{0.9\ln\left(1 + \frac{\varepsilon_c^2}{(\varepsilon'_c)^2}\right)}{\beta_1\left(\frac{\varepsilon_c}{\varepsilon'_c}\right)}, \tag{31}$$

$$c_* = \frac{A_s f_s + A_f f_f - A'_s f'_s}{\gamma f'_c \beta_1 b}, \tag{32}$$

— Calculation of bending moment bearing capacity of beam cross section with fiber:

$$M = A_s f_s\left(d - \frac{\beta_1 c}{2}\right) + 0.85 A_f f_f\left(h - \frac{\beta_1 c}{2}\right) + A'_s f'_s\left(\frac{\beta_1 c}{2} - d'\right). \tag{33}$$

Related to this bearing capacity ($M$), bearing load of beam with fiber has been found from formula (2):

$$P = \frac{6M}{L}. \tag{34}$$

In Table 3, bearing capacity that was computed by formula (4), bearing load that was computed by formula (9); depth of neutral axis ($c$); formulas of (RB1, RB2, RB3, RB4) specimens that were covered with $n = 1, 2, 3, 4$ layers fiber (10–34) and bearing capacities ($M$), bearing loads ($P$), depth of neutral axis ($c$) that were computed by appropriate algorithm have been given.

Table 3a. Analytical computation results of bearing capacities, bearing loads and character other parameters of specimens

| Specimen Name | $M$ (kNm) | $\left(\frac{M_0 - M_i}{M_0} \cdot 100\right)$ $i = 1,2,3,4$ | $P$ (kN) | $\left(\frac{P_0 - P_i}{P_0} \cdot 100\right)$ $i = 1,2,3,4$ | $c$ (m) | $\varepsilon_c$ | $\phi = \frac{\varepsilon_c}{c}$ |
|---|---|---|---|---|---|---|---|
| RB0 | 2.30 | — | 30.67 | — | 0.032 | 0.003 | 93.75 |
| RB1 | 3.48 | 51.30 | 46,48 | 51.54 | 0.034 | 0.003 | 88.23 |
| RB2 | 4.06 | 76.52 | 54.18 | 76.65 | 0.041 | 0.003 | 73.17 |
| RB3 | 4.56 | 98.26 | 60.86 | 98.43 | 0.045 | 0,003 | 66.66 |
| RB4 | 4.81 | 109.13 | 64.14 | 109.12 | 0.049 | 0.003 | 61.22 |

## 6. Analysis with numerical finite element method

Concrete stress element of reinforced concrete beam specimen after separation into 4, 4, 4 pieces in $x$, $y$, $z$ direction with finite elements that have hexahedral surface by quadratic interpolation, bar element after separation into 4 pieces with 3D dimensional frame finite element by quadratic interpolation and CFRP stress layer with shell finite element that has appropriate thickness for fiber coefficient by quadratic interpolation has been modelled in consideration of linear and nonlinear behaviours under static load [7].

Appropriate analysis results have been given in Table 3b. Related to numerical (FEM) computation results in Table 3b.

Lateral buckling moments of beam specimens with CFRP and without CFRP have been computed with following formulas (35, 36) in linear approach (Fig. 6).

$$M_{cr} = \frac{\pi}{L}\sqrt{EI_y GJ}, \tag{35}$$

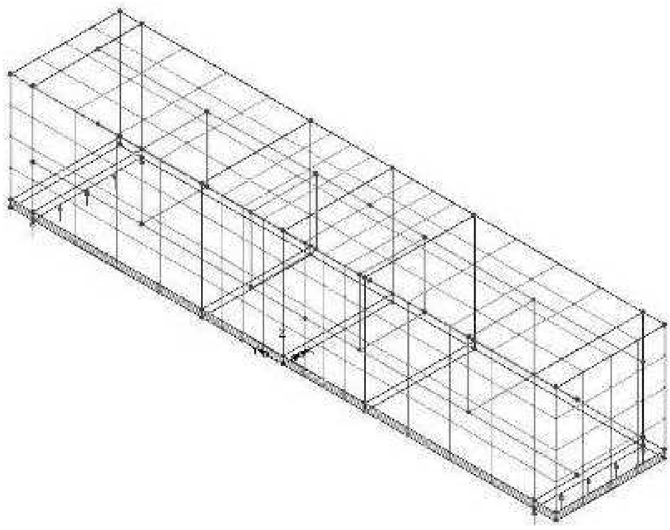


Fig. 5. Specimen numerical (FEM) analysis model [9]

$$I_y = \frac{bh^3}{12}; \qquad J = \frac{hb^3}{3}; \qquad G = 0.4E_c,$$

$$(36) \quad M_{f,cr} = \frac{\pi}{L}\sqrt{(EI)_{ret.}\,GJ}, \quad (EI)_{ret.} = E_f\left(\frac{b_1h_1^3}{12} - \frac{bh^3}{12}\right) + E_c\left(\frac{bh^3}{12}\right).$$

For that specimens lateral buckling moments that was computed with formula (35), (36) are more than appropriate bearing moments.

In the following Table 4, for that specimens, experimental, analytical and numerical (FEM) analysis results and appropriate comparisons have been given.

Table 3b. Numerical computation results of bearing capacities, bearing loads and character other parameters of specimens

| Specimen Name | $M$ (kNm) | $\left(\frac{M_0-M_i}{M_0}\cdot 100\right)$ $i=1,2,3,4$ | $P$ (kN) | $\left(\frac{P_0-P_i}{P_0}\cdot 100\right)$ $i=1,2,3,4$ | $c$ (m) | $\varepsilon_c$ | $\phi=\frac{\varepsilon_c}{c}$ | $u$ (mm) |
|---|---|---|---|---|---|---|---|---|
| RB0 | 2.04 | — | 27.20 | — | 0.032 | 4.76 | 0.014 | 2.88 |
| RB1 | 3.70 | 81.37 | 49.20 | 80.88 | 0.034 | 6.08 | 0.017 | 1.69 |
| RB2 | 3.97 | 94.60 | 53.00 | 94.85 | 0.041 | 2.49 | 0.060 | 1.67 |
| RB3 | 4.56 | 123.52 | 60.80 | 123.52 | 0.045 | 1.28 | 0.002 | 0.99 |
| RB4 | 4.80 | 135.29 | 64.00 | 135.29 | 0.049 | 1.50 | 0.003 | 0.82 |

Table 4. Table of comparison of bearing capacities of specimens that have been obtained by experimental, analytical and numerical (FEM) analysis

| Specimen Name | Experimental results | | | | | | Analytical results | | | | Numerical (FEM) results | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $M$ (kNm) | $P$ (kN) | $\varepsilon_c$ $/10^{-3}$ | $\varepsilon_f$ $/10^{-3}$ | $\phi/10^{-3}$ (Rad/m) | $u$ (mm) | $M$ (kNm) | $P$ (kN) | $\varepsilon_c$ | $\phi$ (Rad/m) | $M$ (kNm) | $P$ (kN) | $\varepsilon_f$ $/10^{-4}$ | $\phi$ (Rad/m) | $u$ (mm) |
| RB0 | 2.29 | 30.54 | 0.22 | 0.42 | 6.875 | 2.92 | 2.300 | 30.67 | 0.003 | 93.75 | 2.04 | 27.20 | 4.76 | 0.014 | 2.88 |
| RB1 | 3.47 | 46.30 | 0.49 | 0.56 | 14.41 | 1.69 | 3.486 | 46.48 | 0.003 | 88.23 | 3.70 | 49.20 | 6.08 | 0.017 | 1.69 |
| RB2 | 4.02 | 53.69 | 0.47 | 0.24 | 11.46 | 1.71 | 4.063 | 54.18 | 0.003 | 73.17 | 3.97 | 53.00 | 2.49 | 0.060 | 1.67 |
| RB3 | 4.50 | 60.09 | 0.41 | 0.12 | 9.11 | 1.00 | 4.565 | 60.86 | 0.003 | 66.66 | 4.56 | 60.80 | 1.28 | 0.002 | 0.99 |
| RB4 | 4.80 | 64.03 | 0.30 | 0.14 | 6.12 | 0.83 | 4.811 | 64.14 | 0.003 | 61.22 | 4.80 | 64.00 | 1.01 | 0.003 | 0.82 |

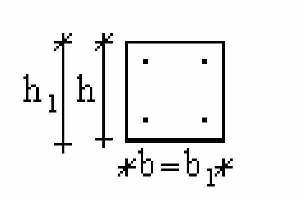



Fig. 6. Cross section of CFRP layered beam

## 7. Conclusions

15 reinforced concrete specimens without CFRP and with suitable for 1, 2, 3, 4 layers CFRP (CF-130) have been tested towards bending and following conclusions based on experimental. Analytical, numerical investigation can be drawn:

— CFRP strengthened reinforced concrete beams increased failure load and moment approximately 70-120 percent depending on the CFRP layer's number.

— For getting maximum fruitfulness of CFRP strengthening, it is necessary fitting observance direction for covering specimens.

— Specimens without CFRP have behaved in a ductile, CFRP strengthened specimens show a brittle behaviour.

— For use full capacity of CFRP, applying an anchorage system may be necessary.

— During testing de-bending type of failure have been dominant (the rupture of CFRP plates is not accurate)

— Research show up that experimental and numerical analysis results are suitable with analytical analysis results by ACI codes.

— For the application traditional known analysis programs for the analysis CFRP strengthened structures may be used the next procedure: a) necessary define load (moment) capacity of the CFRP strengthened structure elements as shown above: b) use equivalent reinforced concrete section appropriate to the CFRP strengthened structure elements; c) this equivalent reinforced concrete section parameters are used as input data for the known analysis programs.

## REFERENCES

[1] ACI, Committee 440, Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Concrete Structures, 26 October 2001.

[2] Ahmed, O., D. V. Gemert, L. Vandewalle. Improved Model for Plate-end Shear of CFRP Strengthened RC Beams. *Cement and Concrete Composites*, **23** (2001), 3–19.

[3] Saadatmanesh, H. An. W., M. R. Ehsani. RC Beams Strengthened with FRP Plates II: Analysis and Parametric Study. *Journal of Structural Engineering*, ASCE, **117** (1991), No. 11, 3417–3433.

[4] Buyukozturk, O., B. Hearing. Failure Behaviour of Pre-cracked Concrete Beams Retrofitted with FRP. *Journal of Composites for Construction,* **2** (1998), No. 3, 138–144.

[5] Dong, Y., M. Zhao, F. Ansari. Failure Characteristics of Reinforced Concrete Beams repaired with CFRP Composites, Proceeding of the third international conference on composites in infrastructure, 2002, California, San Francisco, ICCI, 126–40.

[6] Hata, V. S., Ganga Rao, P. V. Vijay. Bending Behaviour of Concrete Beams Wrapped with Carbon Fabric. *Journal of Structural Engineering*, New York, ASCE, **124** (1998), No. 1, 3–10.

[7] Kasimzade, A. A. Finite Element Method: Foundation and Application to Earthquake Engineering (is included education and finite element analysis programs CD), Istanbul, Beta Publication, (First edition 1997) Second edition, p. 827, 2005 (ISBN 975-511-379-7). `http://www.mathworks.com/support/books/book10424.html`

[8] Klaiber, F. W., T. J. Wipf, B. J. Kempers. Repair of Damaged Prestressed Concrete Bridges Using CFRP, Proceedings of the 2003 Mid-Continent Transportation Research Symposium, Iowa, Ames, 2003.

[9] LUSAS, Finite Element Modelling user manual, 2005.

[10] Mirmiran, A., M. Shahawy, A. Nanni, V. Karbhari. Bonded Repair and Retrofit of Concrete Structures Using FRP Composites: Recommended Construction Specifications and Process Control Manual, NCHRP 514, Transportation Research Board, National Academy of Sciences, Washington D.C, 2004.

[11] Namboorimadathil, S. M., J. G. Tumialan, A. Nanni. Behaviour of RC T-Beams Strengthened in the Negative Moment Region with CFRP Laminates, ICCI 2002, CA, San Francisco, June 10–12, 2002.

[12] Rahimi, H. A. Hutchinson. Concrete Beams Strengthened with Externally Bonded FRP Plates. *J. Compos. Constr.*, **5** (2001), No. 1, 44–56.

[13] Ramos, G., J. R. Casas, A. Alarcon. Repair and Strengthening of Segmental Bridges Using Carbon Bibers, *Engineering Structures*, **26** (2004), No. 5, 609–618.

[14] Teng, J. G., J. F. Chen, S. T. Smith, L. Lam. FRP-strengthened RC Structures. New York: Willey, 2002.

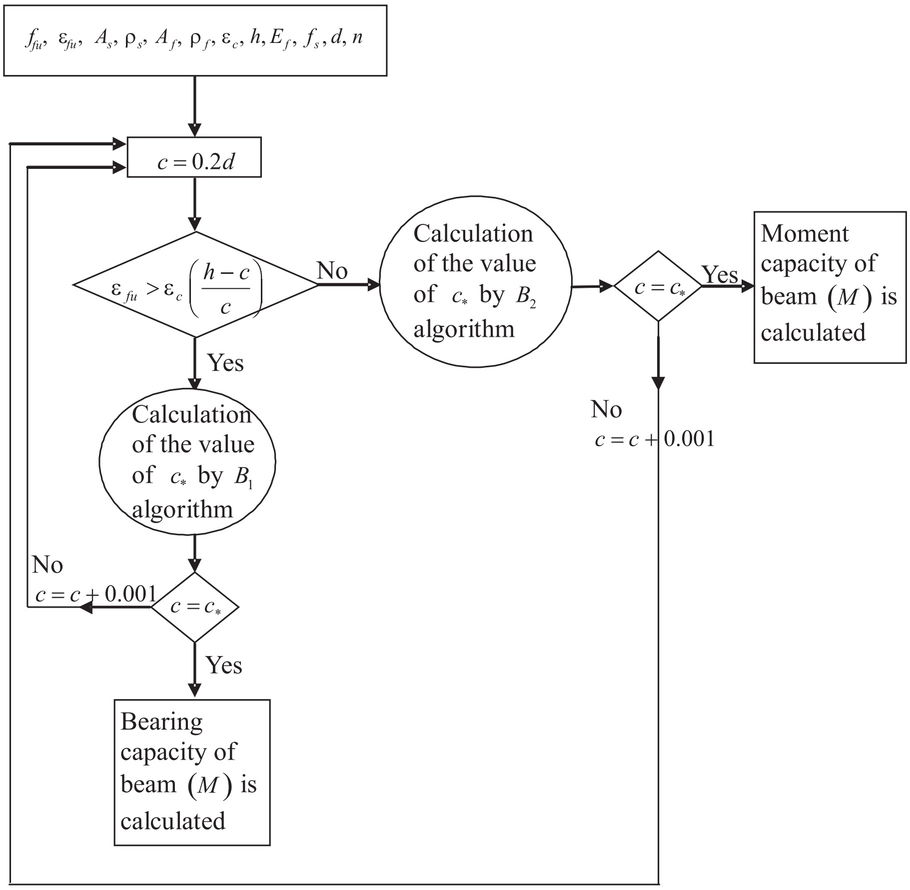
[15] THOMSEN, H., E. SPACONE, S. LIMKATANYU, G. CAMATA. Failure Mode Analyses of Reinforced Concrete Beams Strengthened in Flexure with Externally Bonded Fiber Reinforced Polymers. *J. Compos. Constr.*, **8** (2004), No. 2, 123–131.

[16] WABO MBRACE. Composite Strengthening System Engineering Guidelines, www.mbrace.com.

**Nomenclature**

| | |
|---|---|
| $A_s$: | Area of steel (compression) |
| $A'_s$: | Area of steel (tension) |
| $b$, $h$: | Section dimensions |
| $A_{frp}$: | FRP cross-sectional area |
| $M$: | Moment capacity of beam |
| $M_{cr}$: | Critical moment of beam |
| $M_u$: | Factored moment at a section |
| $c$: | Neutral axis depth (estimated) |
| $c_*$: | Neutral axis depth (calculated) |
| $f_c$: | Compressive concrete strength |
| $f'_c$: | Design concrete strength |
| $d$: | Effective depth |
| $d'$: | Depth to the centroid of the compression steel |
| $d_p$: | Depth to the centroid of the FRP plate |
| $f_{cv}$: | Stress in concrete |
| $f_r$: | Ultimate strength in steel |
| $f_s$: | Stress in tension steel (tension) |
| $f'_s$: | Stress in tension steel (compression) |
| $f_{fu}$: | Design strength of the FRP material |
| $f_{yd}$: | Design yield strength of steel |
| $f_{yk}$: | Characteristic yield stress of steel |
| $J$: | Torsion constant |
| $P$: | Axial load |
| $f_y$: | Yield strength of steel |
| $\varepsilon_p$: | Stress in FRP plate |
| $\varepsilon_{cu}$: | Ultimate concrete compressive strain |
| $\varepsilon_s$: | Stress in tension steel |
| $\varepsilon_r$: | Ultimate strain in steel |
| $\varepsilon_y$: | Yield strain in tension steel |
| $\varepsilon_{fu}$: | Ultimate tensile strain in FRP plate |
| $t_f$: | Thickness of FRP plate |
| $u$: | Displacement |
| $\rho_m$: | Mechanic reinforcement ratio |

**Appendices**




Calculation algorithm of bending moment capacity of beam cross section with CFRP