

SOLID MECHANICS

TRANSVERSE EARTQUAKE-INDUCED VIBRATIONS OF A BURIED PRESSURE PIPELINE INCLUDING FLUID-STRUCTURE INTERACTION

SILVIA PETKOVA, DIMITAR KISLIAKOV

*University of Architecture, Civil Engineering and Geodesy,
1, Hristo Smirnenski Blvd, 1046 Sofia, Bulgaria,
e-mails: silvia.petkova@mail.bg, kiss_fhe@uacg.bg*

YORDAN YORDANOV

*Faculty of Mathematics and Informatics,
5, James Bourchier Blvd, 1164 Sofia, Bulgaria,
e-mail: iordanov@fmi.uni-sofia.bg*

[Received 07 February 2011. Accepted 29 August 2011]

ABSTRACT. In the present work, a theoretical model of the dynamic response of a buried fluid-conveying pressure pipeline to a transverse earthquake excitation is developed. The structural model of the buried pipe corresponds to the type implemented in hydropower systems. It is based on the model for dynamic response of a buried continuous beam on elastic foundation. The seismic excitation is considered as a horizontal shear wave (SH-wave) propagating along the pipe axis. The fluid flow in the pipe is a steady-state one, i.e. no hydraulic transients are accounted for. Furthermore, the earthquake-induced interaction between the vibrating pipe and the flowing liquid inside it is analyzed based on a simplified model of flow-induced vibrations of a fluid-conveying pipe developed by Paidoussis and other authors. The pipe motion governing equation under kinematic excitation has been formulated taking into account all the mentioned interactions. A general closed-form solution of this equation is proposed with separate derivation of the complementary and the particular solutions. A numerical example with real system data has been solved as implementation of the proposed computational procedure for particular boundary conditions and excitation parameters. Finally, conclusions are drawn, and some following tasks for future research are formulated, as well.

KEY WORDS: buried pipe, surrounding soil – pipe – flowing fluid interaction, seismic excitation, closed form solution.

1. Introduction

Buried pipelines are part of the so-called lifelines which play a vital role as infrastructure components conveying and/or distributing energy, fluids, oil products and gas in present-day's world. The pipelines in particular allow conveying water, fossil liquid fuels and liquid gas over long distances. Pressure pipelines also are often important components of large industrial facilities and hydropower systems.

There are different types of pressure pipelines depending on their function, construction material, pressure of the conveyed fluid, position relatively to the ground surface (underground, buried, above ground), etc.

In the following, buried pressure pipelines (penstocks) are considered as components of hydropower systems. They convey water and are constructed in a trench with backfill after the placement. Such pipelines are subjected to various dynamic impacts and loads during their operation like earthquakes, hydraulic transients, traffic, etc. However, only seismic excitation is further considered in this work.

Deterministic approach is applied to the response analysis of a buried pressured pipe during earthquake. It consists of the following steps: description of the physical model; formulation of an appropriate mathematical model of the set of physical phenomena with the corresponding governing equations and their solution.

Hereafter, some already existing mathematical models, governing various aspects of the complex fluid–structure and soil–structure interaction phenomena during seismic excitation, are used and further re-arranged in this work with an appropriate justification. Hence, no particular attention is paid to the principles and methods of construction of the continuous mathematical model.

2. Theoretical model of the surrounding soil – pipe interaction

Destructive movements occur in the pipe–soil system as a result of the seismic vibrations travelling in the form of body and surface waves in the earth crust. They can spread over the soil region through which the pipeline passes, including the pipe structure itself.

Various mathematical models have been developed for describing the interaction between the pipe and the surrounding soil continuum, including some nonlinear effects due to imperfections of the contact at the contact surface as well as the possible sliding mechanism. Special attention has also been paid to the modelling of an active fault crossing. There are numerous studies

on the impact of different types of seismic waves on a buried pipe as well, depending on the parameters of the pipe and the surrounding soil. An extensive structured survey of the works in this field to the current authors' knowledge has been presented in the review report [9, 10].

More recently, intensive research activities have been further carried out on different aspects of the seismic response of the buried pipelines. A detailed description of possible earthquake-induced failure mechanisms and some structural measures to prevent them is presented in [6]. A comparison between analytical and numerical solution approaches in both linear and non-linear formulation to the problems of blast and earthquake impact on a buried pipeline has been carried out in [7] and a series of works by the same authors. Numerical domain discretization methods have been applied to the solution of the governing equations of motion, and different software packages have been compared in the frame of an extensive parameter study of different transient impacts on a buried pipeline structure. A special FE modelling approach using line elements incorporating inelastic cubic formulation has been developed for the earthquake response analysis of a buried pipeline [8]. The nonlinear Winkler Foundation model has been adopted for the soil-pipeline interaction, in which the interactive behaviour is represented by nonlinear discrete soil springs. Various pipe and soil parameters have been accounted for in the study. The obtained results conform in general with the ones of the response spectrum analyses, where the maximum spectral accelerations at the maximum amplifications are in general high in all components of the used Kobe earthquake record.

However, the earthquake-induced pipe-fluid interaction has not been considered in the very few mentioned yet representative examples. To the authors' knowledge, there is still no theoretical model of this phenomenon developed so far.

Special attention is paid to the article [2] for the purpose of this study, which considers the interaction between a continuous pipeline and a homogeneous soil domain during propagation of a seismic shear wave (S-wave) along the pipeline axis. A perfect contact is assumed between the soil and the pipe, and the theoretical model of the pipe is based on the beam-on-Winkler-foundation approach. The pipe-soil interaction is modelled by means of springs and dampers continuously distributed along the contact surface. They are excited at their supports by the earthquake-induced free field displacements and they transmit this kinematic excitation further to the pipe, thus producing stresses and strains. The proposed lateral response model is shown in Fig. 1.

The governing differential equation of the pipe motion, with respect to the unknown function of the lateral displacement $u(x, t)$ of the pipe axis, is:

imposed tension and pressurization effects are either absent or neglected, the equation of motion of the pipe in Fig. 2 takes the particularly simple form:

$$(2) \quad EJ \frac{\partial^4 u(x, t)}{\partial x^4} + MU^2 \frac{\partial^2 u(x, t)}{\partial x^2} + 2MU \frac{\partial^2 u(x, t)}{\partial x \partial t} + (M + m) \frac{\partial^2 u(x, t)}{\partial t^2} = 0,$$

where EJ is the flexural stiffness of the pipe; M is the mass of fluid per unit length, flowing with a steady flow velocity U ; m is the mass of the pipe per unit length, and u is the lateral deflection of the pipe; x and t are the axial coordinate and time, respectively [1].

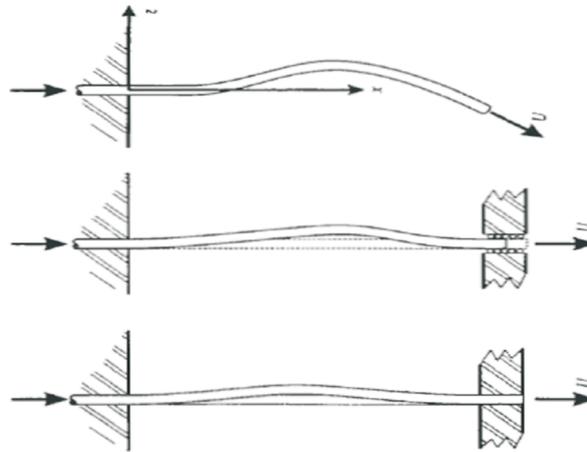


Fig. 2. Pipe conveying fluid with different boundary supporting conditions

The articles [11, 12, 13, 14, 15, 16] deal with the dynamic stability of continuous pipes conveying non-compressible, inviscid and heavy liquid. The pipes lie on elastic supports of Winkler type. Vibrations of the complicated three-component mechanical system are caused during the liquid escaping from the right end of the pipe. The governing equation of pipe motion of such type given in the above mentioned works and based on [1] is:

$$(3) \quad EJ \frac{\partial^4 u(x, t)}{\partial x^4} + MU^2 \frac{\partial^2 u(x, t)}{\partial x^2} + 2MU \frac{\partial^2 u(x, t)}{\partial x \partial t} + (M + m) \frac{\partial^2 u(x, t)}{\partial t^2} + Ku(x, t) = 0.$$

The first four members of Eq. (3) match the ones in Eq. (2) by Paidoussis [1], which describe the interaction between the pipe and the flowing liquid

inside it. Furthermore, the interaction between the pipe and the Winkler elastic support is taken into account by means of the last member in Eq. (3), where K is a stiffness coefficient of the soil foundation. The interpretation of K is “a local foundation vertical response due to local unit displacement of the pipe” [15].

The pipe – fluid – elastic foundation interaction, in cases of different boundary conditions and supporting schemes, is relatively well studied in the above mentioned series of works in terms of critical flutter velocity of the flow.

However, the proposed model of interaction cannot directly be applied to the case of seismic excitation because as it was already mentioned, in fact this impact is a kinematic excitation at the supports of the mechanical model.

4. Theoretical model of the surrounding soil – flowing fluid – buried pipe interaction

The aim of the present study is to analyze the response of a buried pressure pipe as a component of the complex soil–pipe–fluid mechanical system during seismic excitation. In the study below, an attempt is made at modelling the earthquake-induced vibrations of the buried pipe with flowing liquid inside while taking into account the complex interactions occurring between the components of the considered system.

When the theoretical model of the investigated dynamic response was developed, the following assumptions were made:

- the flowing fluid is non-compressible and in view of the fact that the investigated pipeline is considered as hydropower penstock, the fluid inside is assumed to be water;
- the water flow is a steady–state one;
- the mechanical model of the pipeline is assumed to be continuous in the sense of a continuous pipe barrel with constant cross-section characteristics between two supporting base blocks;
- the pipe’s geometrical and mechanical properties are known;
- the soil is assumed to be homogeneous, isotropic and linear elastic with material damping of frequency independent hysteretic type;
- the contact between the pipe and the surrounding soil is perfect in the sense that there is no relative movement (i.e. slip) at the pipe – soil interface;
- the seismic excitation is considered as harmonic function of time and an axial coordinate defining a seismic S-wave travelling along the pipeline, causing lateral displacements only;

- the acceleration, velocity and displacement time histories of two points along the wave's propagation path are assumed to differ only by time lag;
- the lateral response of the pipe is examined in terms of unknown transversal displacement represented by the function $u(x, t)$ in Eq. (4).

The soil-pipe interaction is modelled by continuously distributed springs and dampers which are excited at their supports by the kinematic disturbance of an earthquake nature. They transmit this kinematic excitation further to the pipe. As a result of this excitation in the considered case of steady-state flow, additional forces of hydrodynamic interaction appear and cause pipe loading, respectively. The final aim of the present study is to investigate and model the dynamic pipe response and to determine the stresses and strains induced by the above mentioned phenomena.

The model of the buried pipe was used presented in [2], and the water flow inside the pipe was taken into account as well for the formulation of the governing equation Eq. (4) for the analyzed model of the system surrounding soil – buried pressure pipe – flowing water inside it is subjected to a transverse kinematic seismic excitation. The derivation of Eq. (4) was based on the earthquake-induced vibration of the buried pipe as continuous beam on elastic Winkler foundation (1), to which the corresponding terms of Eq. (2) were further added taking into account the dynamic fluid–structure interaction. It can be clearly seen that the remaining terms on the left-hand side of Eq. (2) governing the fluid–structure interaction are also related to the transverse beam–like movement of the pipe for some particular boundary conditions.

In this sense, the motion governing equation of a buried pressure pipe conveying water under seismic excitation can be written in the form:

$$(4) \quad EJ \frac{\partial^4 u(x, t)}{\partial x^4} + MU^2 \frac{\partial^2 u(x, t)}{\partial x^2} + 2MU \frac{\partial^2 u(x, t)}{\partial x \partial t} + (M + m) \frac{\partial^2 u(x, t)}{\partial t^2} + C \frac{\partial u(x, t)}{\partial t} + Ku(x, t) = C \frac{\partial u_g(x, t)}{\partial t} + Ku_g(x, t),$$

where EJ is the flexural rigidity of the pipe; M is the water mass per unit length flowing with a steady flow velocity U ; m is the pipe mass per unit length; C is the material damping of the soil, K is the soil stiffness parameter, obtained from the model of a beam on elastic foundation of Winkler type [4], $u(x, t)$ is the lateral deflection of the pipe; $u_g(x, t)$ is the transverse soil displacement, that describes the kinematic excitation, x and t are the axial coordinate and time, respectively.

In the following, some analysis of the physical meaning of the terms in the equation of motion for the considered case Eq. (4) is performed. The

first term $EJ \frac{\partial^4 u(x,t)}{\partial x^4}$ in Eq. (4) is the flexural restoring force. It is obvious that the second term $MU^2 \frac{\partial^2 u(x,t)}{\partial x^2}$ is associated with centrifugal forces as the fluid flows in curved portions of the pipe upon recalling that $\frac{\partial^2 u}{\partial x^2} \approx \frac{1}{R}$, where R is the local radius of curvature. Similarly, one can re-write the derivative $\frac{\partial^2 u}{\partial x \partial t} = \frac{\partial \theta}{\partial t} = \Omega$, i.e. the local angular velocity is thus represented. Hence, the third term $2MU \frac{\partial^2 u(x,t)}{\partial x \partial t}$ is associated with Coriolis effects of the interaction between the vibrating pipe and the flowing fluid inside it [1]. The fourth term $(M+m) \frac{\partial^2 u(x,t)}{\partial t^2}$ represents the inertial force of the fluid-filled pipe.

The above mentioned second, third and fourth terms occur in a kinematic analysis after applying the theorem of Coriolis to a differential volume of liquid flowing in the simultaneously vibrating pipe under kinematic excitation.

The terms $C \frac{\partial u(x,t)}{\partial t}$ and $Ku(x,t)$ represent forces resulting from the soil-pipe interaction. The coefficient of viscous material damping C applied here is proportional to the motion velocity of the pipe induced by the seismic wave propagating in the Earth crust. The damping force is a linear continuous function of this velocity.

Here, the inertial forces in the ground are not modelled since the governing equation of motion Eq. (4) is formulated for the pipe and not for the surrounding soil continuum.

The input kinematic support excitation is given by the members of the right-hand side of Eq. (4): $C \frac{\partial u_g(x,t)}{\partial t}$ and $Ku_g(x,t)$, where C and K are damping and stiffness soil parameters, and $u_g(x,t)$ is the transverse soil displacement time history over the pipe axis length which has the form:

$$(5) \quad u_g(x,t) = U_g e^{i\omega(t-x/V)}.$$

Here, U_g is the ground displacement amplitude, ω is the circular frequency of the ground excitation, V is the velocity of the seismic wave propagation along the pipeline axis (i.e. apparent wave velocity).

In the assumed excitation model Eq. (5), the damping with time of the seismic excitation is not accounted for. Hence, it would be appropriate not to account for such damping further while analyzing the pipe structural response, either. On the other hand, this leads to a substantial simplification of the mathematical model of the earthquake-induced buried pipe vibrations.

Furthermore, neglecting the soil damping is not quite an unrealistic assumption since the seismic excitation is definitely of short duration. Last but not least, the final result based on this assumption regarding engineering applications would be on the safety side in the sense of the response parameters.

With these assumptions and after dropping the terms accounting for the soil damping, the governing equation Eq. (4) takes the form:

$$(6) \quad EJ \frac{\partial^4 u(x, t)}{\partial x^4} + MU^2 \frac{\partial^2 u(x, t)}{\partial x^2} + 2MU \frac{\partial^2 u(x, t)}{\partial x \partial t} + (M + m) \frac{\partial^2 u(x, t)}{\partial t^2} + Ku(x, t) = Ku_g(x, t),$$

The complete, general solution $u(x, t)$ of Eq. (6) as for any linear differential equation, is the sum of the solution of the homogeneous equation (s.h.e) and a particular solution of the non-homogeneous equation (s.n.e) [3, 5]:

$$(7) \quad u(x, t) = u(x, t)_{s.h.e} + u(x, t)_{s.n.e}.$$

Here, the first step is obtaining the particular solution of Eq. (6). It is particular with respect to the loading function (5), and in this case it is assumed to be written in the form:

$$(8) \quad u(x, t)_{s.n.e} = A.e^{i\omega(t-\frac{x}{V})},$$

where A is the unknown amplitude. It should be emphasized here that this proposed form of the particular solution corresponds to a “running wave” model for the structural response of the pipe. Thus, the nature of both the seismic excitation, Eq. (5) and the pipe response to it can be represented more truly unlike the approach to the particular solution applied in [2].

The unknown amplitude A after substituting the proposed solution Eq. (8) in Eq. (6) as a “trial” one and after some mathematics and differentiation, is found in the form:

$$(9) \quad A = \frac{K}{EJ \frac{\omega^4}{V^4} - MU^2 \frac{\omega^2}{V^2} + 2MU \frac{\omega^2}{V} - (M + m).\omega^2 + K} U_g.$$

Accordingly, the solution of the non-homogeneous equation of Eq. (6) with respect to Eqs (8) and (9) is:

$$(10) \quad u(x, t)_{s.n.e} = \frac{K}{EJ \frac{\omega^4}{V^4} - MU^2 \frac{\omega^2}{V^2} + 2MU \frac{\omega^2}{V} - (M + m).\omega^2 + K} U_g.e^{i\omega(t-\frac{x}{V})}.$$

The second step of the proposed computational approach is obtaining the solution of the homogeneous equation. The homogeneous equation of Eq. (6) is:

$$(11) \quad EJ \frac{\partial^4 u(x, t)}{\partial x^4} + MU^2 \frac{\partial^2 u(x, t)}{\partial x^2} + 2MU \frac{\partial^2 u(x, t)}{\partial x \partial t} + (M + m) \frac{\partial^2 u(x, t)}{\partial t^2} + Ku(x, t) = 0,$$

The solution of the homogeneous equation (11) by separation of the variables is assumed here to be of the form:

$$(12) \quad u(x, t)_{s.h.e} = u(x) \cdot e^{i\omega t}.$$

Substituting function (12) in Eq.(11) leads to:

$$(13) \quad EJ.u(x)'''' e^{i\omega t} + MU^2.u(x)'' e^{i\omega t} + 2MU(i\omega).u(x)' e^{i\omega t} - (M + m).\omega^2.u(x)e^{i\omega t} + K.u(x)e^{i\omega t} = 0,$$

where the primes denote derivatives with respect to the axial coordinate x .

In order for Eq.(13) to be correct, the multiplier before the exponent must be zero, i.e.:

$$(14) \quad EJ.u'''' + MU^2.u'' + 2MU(i\omega).u' + (K - (M + m).\omega^2) u = 0.$$

The characteristic equation of Eq. (14) is:

$$(15) \quad EJ.\lambda^4 + MU^2.\lambda^2 + 2MU(i\omega).\lambda + K - (M + m).\omega^2 = 0.$$

The general solution of Eq. (6) after computation of the complex roots λ_k ($k = 1, 2, 3, 4$) of the characteristic Eq. (15), when λ_k are simple roots, as seen by Eq. (23), and according to Eq. (7) and Eq. (10), may be written as:

$$(16) \quad u(x, t) = \sum_{k=1}^4 C_k e^{\lambda_k x} e^{i\omega t} + \frac{K}{EJ \frac{\omega^4}{V^4} - MU^2 \frac{\omega^2}{V^2} + 2MU \frac{\omega^2}{V} - (M + m).\omega^2 + K} U_g e^{i\omega(t - \frac{x}{V})},$$

Furthermore, Eq. (16) has to satisfy the corresponding boundary conditions in the particularly formulated problem.

5. Numerical example

A numerical example is solved as an illustration of the above presented analytical solution with the developed model for the response of a buried pipe conveying water during seismic excitation. For the purposes of this numerical example, a section of a buried steel pipe with length L , located between two concrete anchor blocks, is considered. The pipe is excited by seismic shear wave propagating along its axis with apparent velocity, V . As usual, the pipeline axis is assumed to be parallel to the ground surface, Fig. 3. In the case when the incidence angle of the seismic wave relative to the pipe axis is arbitrary (non-zero), the axial and transversal contributions of the seismic excitation can be obtained by a simple geometric transformation.

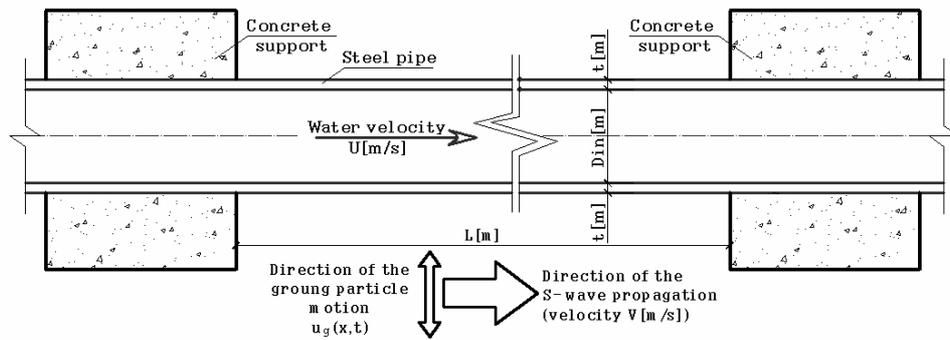


Fig. 3. Lateral response model of buried pipeline conveying water under seismic shear wave propagation (Layout)

Input data for the considered model:

- Material and geometrical characteristics of the steel pipe [20]
 - $r = 7.85 \text{ t/m}^3$ – Density of the steel per unit volume;
 - $E = 2.1 \times 10^8 \text{ kN/m}^2$ – Young's modulus of the steel;
 - $m = 0.249 \text{ t/m}'$ – Mass of the steel pipe per unit length;
 - $EJ = 849309 \text{ kNm}^2$ – Flexural rigidity of the pipe's cross section;
 - $L = 200 \text{ m}$ – Length of the pipe between two concrete anchor blocks;
- Input data for the flowing water
 - $r = 1 \text{ t/m}^3$ – Density of the water per unit volume;
 - $M = 0.785 \text{ t/m}'$ – Mass of the water per unit length;
 - $U = 2.50 \text{ m/s}$ – Velocity of the flowing water;

- Input data for the assumed soil type – type C (“Dense soil” [19])
 - $\gamma = 18 \text{ kN/m}^3$ – Unit weight of the soil [17];
 - $C = 30 \text{ kPa}$ – Coefficient of cohesion of the soil [17];
 - $\phi = 30^\circ$ – Angle of an internal friction of the soil [17];
 - $H = 1.50 \text{ m}$ – Soil cover above the center of the pipeline.

- Input data for the seismic excitation:

The seismic excitation is assumed to be a shear wave propagating along the pipeline axis. According to [19], a realistic value of the propagation velocity for the assumed soil type C is $V = 300 \text{ m/s}$.

The circular frequency of the seismic excitation ω is a parameter of crucial influence on the dynamic response of the considered mechanical model of a fluid-conveying buried pipe. In Seismic Mechanics, different types of earthquakes are defined and analyzed with respect to their characteristic frequency, there are ones with predominant frequency and ones representing a wider frequency content. The solution can be carried out for this frequency by means of the proposed here model in the case with a predominant excitation frequency. The solution should be performed in a way comprising the whole part of the significant frequency content (in the sense of a Fourier amplitude frequency spectrum or response spectrum) when the frequency spectrum of the excitation is much wider, for example by walking through this range of interest with certain frequency step. The solution has to be performed by means of the proposed here computational procedure for each of these discrete frequency values. Finally, conclusions need to be drawn based on the whole set of solutions with respect to the particular conditions and the problem statement.

The predominant circular frequency of the seismic excitation for the considered interaction model here is assumed to be $\omega = 8 \text{ rad/s}$.

A question remains open about the amplitude of the lateral ground displacement function U_g in Eq. (5) as well as about the stiffness of the lateral soil springs, K in Eq. (6), used for modelling the soil – pipe interaction:

- The computational procedure given in EN 1998-1:2004 [19] is applied in the frame of the present solution to obtain the amplitude of the lateral soil displacement U_g . The suggested formula in [19] is:

$$(17) \quad U_g = 0.025 \cdot a_g S \cdot T_C \cdot T_D = 0.25 \cdot 0.27g \cdot 1.15 \cdot 0.60 \cdot 2 = 0.09 \text{ m},$$

where:

- U_g – The maximum ground displacement during seismic action for an assumed elastic acceleration response spectrum of Type 1, shown in Fig. 4;

- $a_g = k_c g = 0.27 \text{ g}$ – The peak design ground acceleration of the soil layer, type A ground, for intensity level IX (as assumed here);
- $T_C = 0.60 \text{ sec}$ – The upper limit of the period of the constant spectral acceleration branch
- $T_D = 2.00 \text{ sec}$ – The value defining the beginning of the constant displacement response range of the spectrum;
- $S = 1.15$ – The soil factor.

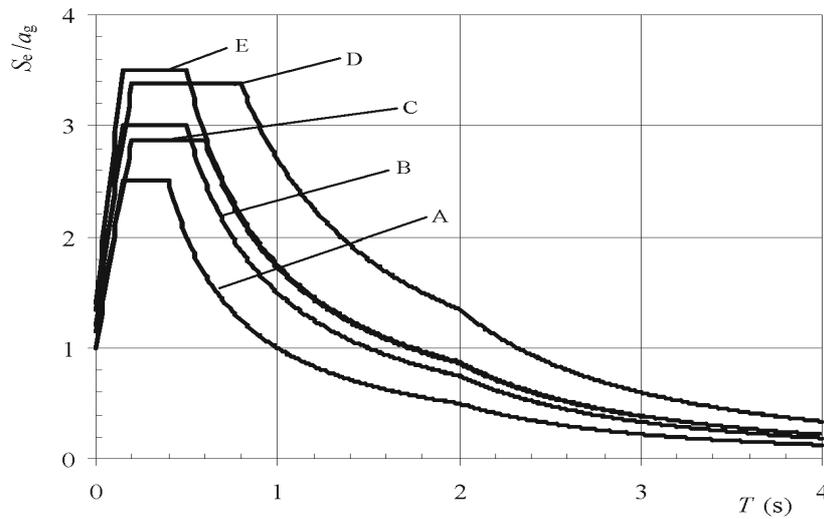


Fig. 4. Recommended Type 1 Elastic response spectra for ground types A to E (5% damping) [19]

- The computational procedure given in [18] has been further applied in this work for calculating the properties of the lateral soil spring stiffness K . They were estimated considering the native soil at the site. Figure 5 shows the idealized representation of the lateral soil spring.

The maximum lateral resistance of the soil per unit length of the pipe can be calculated as:

$$(18) \quad K = N_{ch}c(D_{in} + 2t) + N_{qh}\gamma H(D_{in} + 2t),$$

where N_{ch} is the horizontal bearing capacity factor for clay; N_{qh} is the horizontal bearing capacity factor for sandy soil; γ is the unit weight of soil.

$$N_{ch} = a + bz + \frac{c}{(z + 1)^2} + \frac{d}{(z + 1)^3}; \quad N_{qh} = a + bz + cz^2 + dz^3 + ez^4,$$

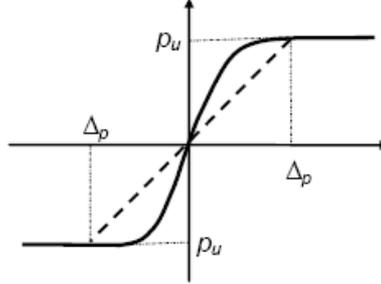


Fig. 5. Idealized representation of lateral soil spring [18]

where $z = \frac{H}{(D_{in} + t)} = \frac{1.51}{1,02} = 1.48$ m; and the parameters a, b, c, d, e are accepted of the data given in Table B2 [18] for clay and sandy clay.

Thus,

$$N_{ch} = 6.752 + 0.065 \cdot 1.48 + \frac{-11.063}{(1.48 + 1)^2} + \frac{7.119}{(1.48 + 1)^3} = 5.52,$$

$$N_{qh} = 4.565 + 1.243 \cdot 1.48 - 0.089 \cdot 1.48^2 + 4.275 \cdot 10^{-3} \cdot 1.48^3 - 9.159 \cdot 10^{-5} \cdot 1.48^4 = 6.22$$

Hence, the maximum transverse soil resistance per unit length of the pipe (soil spring stiffness) is:

$$K = 5.52 \cdot 30 \cdot (1.00 + 2 \cdot 0.01) + 6.22 \cdot 18 \cdot 1.51 \cdot (1.00 + 2 \cdot 0.01) = 341 \text{ kN/m}.$$

The solution with real data after specifying the material and geometrical characteristics of the considered model of the buried water-conveying pressure pipeline under seismic excitation can be performed.

The characteristic equation Eq. (15) with the real data becomes:

$$(19) \quad 849309\lambda^4 + 4.90\lambda^2 + 31.40i\lambda + 274.82 = 0$$

The complex roots λ_j of Eq. (19) are determined by means of the mathematical software Maple and their values are:

$$(20) \quad \begin{aligned} \lambda_1 &= -0.1131 - 0.1124i, & \lambda_2 &= -0.1117 + 0.1124i, \\ \lambda_3 &= 0.1116 + 0.1124i, & \lambda_4 &= 0.1131 - 0.1124i \end{aligned}$$

The solution of the non-homogeneous equation is computed in the form:

$$(21) \quad u(x, t)_{s.n.e} = 1.235 \cdot U_g e^{i\omega(t - \frac{x}{V})}$$

Hence, the general solution of Eq. (6) is:

$$(22) \quad u(x, t) = (C_1 e^{(-0.1131-0.1124i)x} + C_2 e^{(-0.1117+0.1124i)x} + C_3 e^{(0.1116+0.1124i)x} + C_4 e^{(0.1131-0.1124i)x}) e^{i\omega t} + 1.235 U_g e^{i\omega(t - \frac{x}{V})}$$

and it has to satisfy the corresponding boundary conditions in the particular case.

These boundary conditions of the considered in this study model are:

$$(23) \quad \text{at } x = 0 \left\{ \begin{array}{l} u(0, t) = u_g(0, t) \\ \frac{\partial u}{\partial x}(0, t) = 0 \end{array} \right. ,$$

$$(24) \quad \text{at } x = L \left\{ \begin{array}{l} u(L, t) = u_g(L, t) \\ \frac{\partial u}{\partial x}(L, t) = 0 \end{array} \right. .$$

These mathematical formulations of boundary conditions correspond to the fixing of the pipe at both ends in concrete anchor blocks, i.e. fix clamped support conditions at the ends of the analyzed pipe segment with length L .

It is assumed that the concrete base blocks do not deform since they are solid elements with high stiffness and the displacements there are equal to those in the ground, i.e. the displacements of the clamped pipe ends at the concrete blocks coincide with the earthquake-induced ones of the ground.

The following system of equations for the unknown integration constants in Eq. (22), after satisfying the corresponding boundary conditions in Eqs (23) and (24), results in the form:

$$\begin{aligned}
(25) \quad & \left. \begin{aligned}
& C_1 + C_2 + C_3 + C_4 + 0.235U_g = 0 \\
& (-0.1131 - 0.1124i)C_1 + (-0.1117 + 0.1124i)C_2 + \\
& +(0.1116 + 0.1124i)C_3 + (0.1131 - 0.1124i)C_4 + 0.0329iU_g = 0 \\
& C_1e^{(-22.62-22.48i)} + C_2e^{(-22.34+22.48i)} + \\
& +C_3e^{(22.32+22.48i)} + C_4e^{(22.62-22.48i)} + 0.235U_ge^{-5.33i} = 0 \\
& (-0.1131 - 0.1124i)C_1e^{(-22.62-22.48i)} + \\
& +(-0.1117 + 0.1124i)C_2e^{(-22.34+22.48i)} + \\
& +(0.1116 + 0.1124i)C_3e^{(22.32+22.48i)} + \\
& +(0.1131 - 0.1124i)C_4e^{(22.62-22.48i)} + \\
& +0.0329iU_ge^{-5.33i} = 0.
\end{aligned} \right\}
\end{aligned}$$

The unknown integration constants for the estimated maximal value of the ground displacement $U_g = 0.09$ m, Eq. (17), are derived as follows:

$$\begin{aligned}
(26) \quad & C_1 = 0.00237 - 0.0107i \\
& C_2 = 0.0025 + 0.0107i \\
& C_3 = -1.421 \cdot 10^{-12} + 1.7 \cdot 10^{-12}i \\
& C_4 = 2.046 \cdot 10^{-12} + 3.320 \cdot 10^{-12}i
\end{aligned}$$

Hence, the general solution (7) of Eq. (6) by substituting the complex constants (26) in Eq. (22), is obtained in the form:

$$\begin{aligned}
(27) \quad & u(x, t) = [(-0.0237 - 0.0107i)e^{(-0.1131-0.1124i)x} \\
& + (0.0025 + 0.0107i)e^{(-0.1117+0.1124i)x} \\
& + (-1.421 \cdot 10^{-12} + 1.7 \cdot 10^{-12}i)e^{(0.1116+0.1124i)x} \\
& + (2.046 \cdot 10^{-12} + 3.320 \cdot 10^{-12}i)e^{(0.1131-0.1124i)x}]e^{8it} \\
& + 1.235U_ge^{8i\left(t-\frac{x}{300}\right)}
\end{aligned}$$

The dynamic analysis of the developed model is performed in the complex domain (plane), and the proposed forms of the complementary and particular solutions Eqs. (5) and (12) are also complex functions. Thus, the general

solution is a complex function as well. After some calculus, the general solution Eq. (27) gets the form:

$$\begin{aligned}
 u(x, t) = & -0.0237e^{-0.1131x} \cos(-0.1124x + 8t) \\
 & + 0.0107e^{-0.1131x} \sin(-0.1124x + 8t) \\
 & + i.[-0.0107e^{-0.1131x} \cos(-0.1124x + 8t) \\
 & - 0.0236.e^{-0.1131x} \sin(-0.1124x + 8t)] \\
 & + 0.0025e^{-0.1117x} \cos(0.1124x + 8t) \\
 & - 0.0107e^{-0.1117x} \sin(0.1124x + 8t) \\
 & + i.[0.0107e^{-0.1117x} \cos(0.1124x + 8t \\
 & + 0.0025e^{-0.1117x} \sin(0.1124x + 8t)] \\
 (28) \quad & - 1.421 \cdot 10^{-12} e^{0.1116x} \cos(0.1124x + 8t) \\
 & - 1.7 \cdot 10^{-12} e^{0.1116x} \sin(0.1124x + 8t) \\
 & + i.[1, 7 \cdot 10^{-12} e^{0.1116x} \cos(0.1124x + 8t) \\
 & - 1.421 \cdot 10^{-12} e^{0.1116x} \sin(0.1124x + 8t)] \\
 & + 2.046 \cdot 10^{-12} e^{0.1131x} \cos(-0.1124x + 8t) \\
 & - 3.32 \cdot 10^{-12} e^{0.1131x} \sin(-0.1124x + 8t) \\
 & + i.[3.32 \cdot 10^{-12} e^{0.1131x} \cos(-0.1124x + 8t) \\
 & + 2.046 \cdot 10^{-12} e^{0.1131x} \sin(-0.1124x + 8t)] \\
 & + 0.11115 \cos(-0.0267x + 8t) + i.[0.11115 \sin(-0.0267 + 8t)]
 \end{aligned}$$

The analysis of the solution continues with the separation of the real part of the Eq. (28) from the imaginary part since only the real part corresponds to the physical sense of the modelled phenomena.

The real part of Eq. (28) is:

$$\begin{aligned}
 \Re\{u(x, t)\} = & -0.0237e^{-0.1131x} \cos(-0.1124x + 8t) \\
 & + 0.0107e^{-0.1131x} \sin(-0.1124x + 8t) \\
 & + 0.0025e^{-0.1117x} \cos(0.1124x + 8t) \\
 & - 0.0107e^{-0.1117x} \sin(0.1124x + 8t) \\
 (29) \quad & - 1.421 \cdot 10^{-12} e^{0.1116x} \cos(0.1124x + 8t) \\
 & - 1.7 \cdot 10^{-12} e^{0.1116x} \sin(0.1124x + 8t) \\
 & + 2.046 \cdot 10^{-12} e^{0.1131x} \cos(-0.1124x + 8t) \\
 & - 3.32 \cdot 10^{-12} e^{0.1131x} \sin(-0.1124x + 8t) \\
 & + 0.11115 \cos(-0.0267x + 8t)
 \end{aligned}$$

As an example, the evaluation of Eq. (29) has been carried out for a few fixed moments of time $t = 1$ s, 5 s, 10 s, 20 s, 30 s. The graphics shown below in Fig. 6 depicts as illustration the transverse displacements of the points of

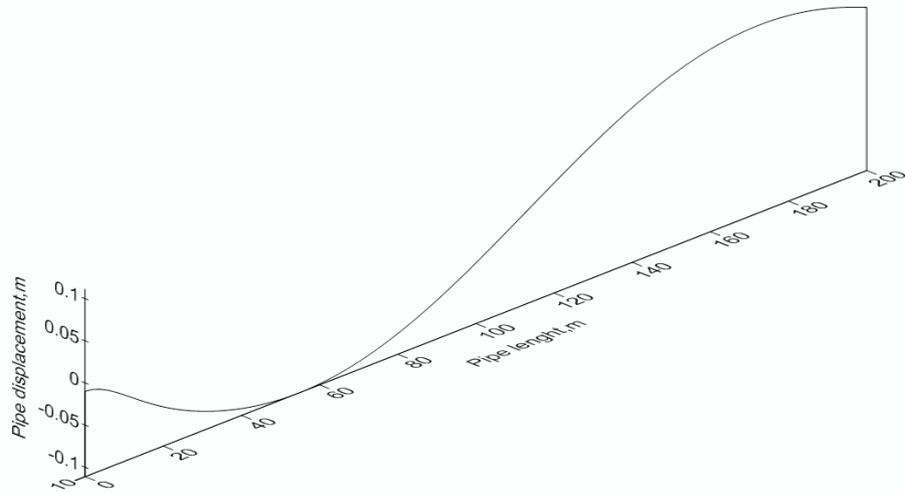


Fig. 6. Transverse displacements of the points of the pipe axis at time moment $t = 10$ s

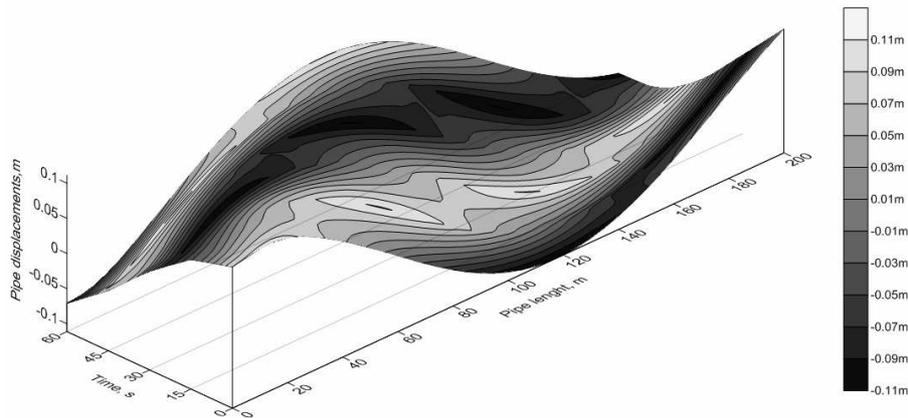


Fig. 7. Transverse displacements of the pipe axis points caused by excitation of 60 s

the pipe axis with coordinates $x_n = n \cdot \Delta x \begin{cases} n = 1, 2, 3, \dots \\ \Delta x = 1 \text{ m} \end{cases}$ for $t = 1$ s, where the values between them are defined by linear interpolation.

Here, the question remains open about the evaluation of the extreme values of the function (29). In the particular case, this problem has been solved numerically. The step length over the pipe axis was $\Delta x = 0.50$ m, and the time step was $\Delta t = 0.05$ s.

The results from this calculation are shown in Fig. 7. The obtained maximum absolute value of the pipe axis displacement was 0.11 m.

Conclusion

To our knowledge, for the first time a theoretical model is presented of the dynamic response of a buried fluid-conveying pressure pipeline under seismic excitation. The proposed here closed-form solution allows analytical evaluation of the pipeline response for particular boundary conditions and predominant frequency of the imposed kinematic excitation of SH-wave type in the time domain. Although the governing equation of the problem has been formulated, based on existing models describing particular interaction phenomena, the related carried out physical analysis results in an attempt for revealing all involved interaction phenomena.

We would suggest as a next task for the future research the development of the model of the axial earthquake-induced vibrations of a buried pressure pipe as well as incorporation of an appropriate model of the damping effects of both excitation and dynamic structural pipeline response.

REFERENCES

- [1] PAIDOUSSIS, M. P. Fluid-structure Interactions, ISBN 0-12-544360-9, Academic Press, 1998
- [2] MAVRIDIS, G., K. PITILAKIS. Axial and Transverse Seismic Analysis of Buried Pipeline, Paper No. 1605, Proc. 11th WCEE, Elsevier Science Ltd., 1996.
- [3] CHEN, W., C. SCAWTHORN. Earthquake Engineering Handbook, ISBN 0-8493-0068-1, SRS Press LLC, 2003
- [4] KARAMANSKI, T., T. GANEV, T. BOBEV, A. POPOV, N. KAPITANOV, I. BAJCHEV. Structural Mechanics, Sofia, "Technika" 1988, (in Bulgarian).
- [5] HARTMAN, P. Ordinary Differential Equations, New York-London-Sydney, John Wiley & Sons, 1964.
- [6] BALLANTYNE, D. Seismic Vulnerability Assessment and Design of Pipelines. *Journal AWWA*, Vol. **102**, (2010), No. 5, 88–97.
- [7] OLAREWAJU, A. J., N. S. V. KAMESWARA RAO, M. A. MANNAN. Guidelines for the Design of Buried Pipes to Resist Effects of Internal Explosion, Open Trench and Underground Blasts, *EJGE. Bundle J.*, Vol.**15**, (2010), 645–658.
- [8] LEE, D. H., B. H. KIM, H. LEE, J. S. KONG. Seismic Behaviour of a Buried Gas Pipeline under Earthquake Excitations. *Engineering Structures*, Vol. **31** (2009), 1011–1023.

- [9] PETKOVA, S., D. KISLIAKOV. Earthquake-Induced Behaviour of Buried Pressure Pipelines: State-of-the-Art I. Modelling of Physical Phenomena And Methods of Analysis, Int. Conference UACEG 2009: Science & Practice, Sofia, 2009, 31–41.
- [10] PETKOVA, S., D. KISLIAKOV. Earthquake-Induced Behaviour of Buried Pressure Pipelines: State-Of-The-Art II. Computational Procedures for Practical Use, Proc. Int. Conference UACEG 2009: Science & Practice, Sofia, 2009, 43–53.
- [11] LILKOVA-MARKOVA, S., V. AL. DZHUPANOV. Dynamic Stability of Cantilevered Pipes Supported By Additional Structural Spring Supports, Part 1. Short Pipes, Proceedings of the 9th Bulgarian National Congress of Theoretical and Applied Mechanics, September 19–22, 2001, 528–534.
- [12] LILKOVA-MARKOVA, S., V. AL. DZHUPANOV. Dynamic Stability of Cantilevered Pipes Supported By Additional Structural Spring Supports, Part 2. Long Pipes, Proceedings of the 9th Bulgarian National Congress of Theoretical and Applied Mechanics, 2001, 528–534.
- [13] LILKOVA-MARKOVA, S., V. AL. DZHUPANOV. Dynamic Stability of a Clamped Tube, conveying a Fluid and Suspended on a Specific Combined Support, Conference on Acoustics, Sofia, 2001, (in Bulgarian).
- [14] LILKOVA-MARKOVA, S. An Influence of the Elastic Foundation over the Dynamic Stability of Pipes Conveying Fluid in Three Cases of a Supporting, Jubilee Scientific Conference “60 Years of UASG”, Sofia, 2002, 339–344 (in Bulgarian).
- [15] LILKOVA-MARKOVA S., V. AL. DZHUPANOV. Statement Ruses of the Problem on the Dynamic Stability of Cantilevered Pipe Conveying Liquid and Lying on Elastic Foundation. *Journal of Theoretical and Applied Mechanics*, Vol. **33** (2003), 55–72.
- [16] DJONDJOROV P., V. VASSILEV. Dynamic Stability of Viscoelastic Pipes on Elastic Foundations of Variable Modulus. *Journal of Sound and Vibration* **297** (2006), 414–419.
- [17] KOSTOV, V., TR. GERMANOV, K. FILIPOV. Soil Properties Evaluation for Numerical Modelling of Soil-Pipeline Interaction, Volume **1**, Report N 1501-R-004, EQE Bulgaria, Sofia, 1995, 1–62.
- [18] DASH, S., S. JAIN. “Guidelines for Seismic Design of Buried Pipelines”, IITK-GSDMA, Indian Institute of Technology, India, Kanpur, 2007.
- [19] EN 1998–1:2004 “Eurocode 8 – Design of Structures for Earthquake Resistance – Part 1: General Rules, Seismic Actions and Rules for Buildings”, 2004.
- [20] PEJCHEV, P. “Manual on Steel Profiles”, Sofia, “Technika”, 1978 (in Bulgarian).