Theorem on decomposition of eigenstrain and residual biological stress

V. Lokhov, Y. Nyashin

Perm National Research Polytechnic University, 29 Komsomolskii Prospect, 614990, Perm, Russia;
Valery.Lokhov@yandex.ru

Abstract
Residual biological stresses exist apparently in all living systems (human being, animals, plants, et al.). In human organism, these stresses exist in different organs and tissues (bones, brain, embryo, etc). According to Reissner (1931), residual biological stresses can be considered as eigenstresses produced by eigenstrains if these stresses are self-equilibrating. More specifically, the biological strains can be realized as growth strains in a growing organism and remodeling strains arising at remodeling due to change of internal or external conditions. The authors of this paper proved the theorem on decomposition of eigenstrain into impotent eigenstrain (not inducing stress in any point of a body) and nilpotent eigenstrain (not inducing strain in any point of the body). According to the theorem, the residual stress can be found through the nilpotent eigenstrain.

If the eigenstrain is an impotent one, the eigenstress equals zero. For this aim, the eigenstrain must be compatible and appropriate displacements must vanish on the part of the boundary with immovable supports. In the case of multiply connected body, the integral relations must be added.

If the displacements are single-valued and continuous together with derivatives with respect to coordinates up to the third order, the compatibility equations (Saint-Venant, 1861) can be derived. In the more general case, the strain compatibility or the degree of incompatibility of a eigenstrain tensor (and in consequence, the estimation of level of eigenstress or residual stress) can be found through the distance of this in the function space of eigenstrain from the subspace of compatible strain. In given paper, the proofs of discusses results are considered.

In this paper, the tensor of strain is considered as a special case of eigenstrain (Reissner, 1931). The authors proved the theorem on decomposition of eigenstrain existing in a body into two constituents: impotent eigenstrain (not causing stress in any point of a body) and nilpotent eigenstrain (not causing strain in any point of a body). According to this theorem, the thermal stress can be easily found through the nilpotent eigenstrain. If the eigenstrain is an impotent one, the thermal stress vanishes. In this case, the eigenstrain must be compatible. The authors suggest new approach to measure of eigenstrain incompatibility and hence to estimate thermal stresses.

Keywords: Eigenstrain, residual biological stresses, decomposition, impotent eigenstrain, nilpotent eigenstrain, functional space