Description of the nonlinear viscoelastic and thixotropic properties of human blood by the modified model of Maxwell-Gurevich-Rabinovich

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Abstract: Modeling by means of employing the generalized and modified Maxwell-Gurevich-Rabinovich (MGR) model, presented in this study, is based on our experimental data found using Low Shear 30 sinus Contraves rotational viscometer at different regimes of blood shear flow: steady and unsteady – transient and sinusoidal flow. The rheological behaviour of the human blood is characterized by pseudoplasticity, thixotropy and viscoelasticity. According to the MGR model the full strain rate at shear is a superposition of the elastic strain rate, the residual strain rate and the high elastic strain rate. The elastic modulus of shear $G$, the initial coefficient of viscosity $\eta_0$ and the logarithmic modulus of the strain rate $m$ link $\left(\tau, \eta, \dot{\gamma}, \dot{\gamma}\right)$ in functional dependences and characterize the rheological behaviour of blood at the considered flow regimes. Introducing nonlinear dependences of the logarithmic strain modulus $m$ on shear rates $\dot{\gamma}$ for a detailed description of the observed relationships some modifications of the MGR model are proposed. The parameters of the proposed models are found from experimental data and used for blood micro structuring analysis.

Key words: blood thixotropy and viscoelasticity, Maxwell-Gurevich-Rabinovich (MGR) model, initial viscosity coefficient, logarithmic strain modulus, elastic shear modulus

1. Introduction

Alterations of blood flow with time is typical for thixotropic fluids and this phenomenon can be related to the influence of the inner structure on the blood flow. In the case of thixotropy when flow stops, particles connect with each other and the process of destroying and creating new connections is well balanced, while forces of connection are weak. Higher shear rate leads to decrease of the size of the related structural units, corresponding to a less resistance to flow.

When the fluid is with larger shear rates, more macromolecules are destroyed for a unit time. Consequently, thixotropy and viscoelasticity are related to similar time effects, reflecting shear rate effects. From this point of view theories are developed, describing the mechanisms of thixotropy and viscoelasticity by means of unified models. The generalized model of Maxwell-Gurevich-Rabinovich (MGR) for polymer solutions, deduced from the thermo-fluctuation theory by
Rabinovich [1], has been applied in this study to model the rheological properties of human blood. The model is characterized by two constants - initial viscosity coefficient and logarithmic strain modulus. The observed experimental dependences for blood rheological properties at three different regimes of viscometric shear flow: steady shear, time-dependent flow at rectangular and triangular change of shear rates and sinusoidal oscillation flow presented previously [2], are described by the MGR model. For description of the experimental relationships the MGR model is modified, including shear rate dependence of its initial viscosity coefficient and taking into account the shear rate effect for any specific shear regime and range.

2. Materials and methods

2.1. Materials and experimental methods

3-4 ml blood was drawn from a cubital vein between 8 and 10 a.m. and heparinized (15 IU/ml blood). Samples of 1-ml were measured for up to 5 hours after blood sample preparation at 37 °C keeping their native hematocrit, for 11 shear rates ranging from 0.0237 to 128.5 s⁻¹. Estimations of the hematocrit of the same samples were done, using microhematocrit centrifuge TH12 and performing centrifugation for 5 min at 10 000 min⁻¹.

Normal blood conserved with L-12 (300 ml blood/75 ml L-12) and a group of 22 patients (14 women, 8 men, mean age 58.6 ± 11 years) with cerebrovascular disease (CVD) were investigated. Blood sample from a patient (female) with chronic opioid addicts under methadone maintenance therapy (DA) was chosen from a group of 15 chronic opioid addicts under methadone maintenance therapy (DA) (3 women, 12 men, mean age 26.53 ± 7.34 years). Blood was provided by the National Center for Addictions in Sofia. In addition, normal human heparinized blood taken from two healthy persons was centrifuged and its hematocrit was adjusted (reconstituted) within a range of 30% to 70%, i.e. to values of 30%, 40%, 50%, 60% and 70%.

Rheograms of change of the shear stresses ($\tau$) with time at a triangular change of shear rates ($\dot{\gamma}$) were plotted by Rikadenki plotter. Hysteresis was drawn at a triangular change of shear rates $\dot{\gamma} = \alpha t$ for $0 < t < t_{\text{max}}$ - loading and $\dot{\gamma} = \alpha (2t_{\text{max}} - t)$ for $t_{\text{max}} < t < 2t_{\text{max}}$ - relaxation, where a) $\dot{\gamma}_{\text{max}} = 0.0596$ s⁻¹ and $t_{\text{max}} = 30$s; b) $\dot{\gamma}_{\text{max}} = 1.18$ s⁻¹ and $t_{\text{max}} = 30$s.

The steady and dynamic viscosity dependence on shear rate was determined using a rotational viscometer with a firmly set amplitude of ± 30° “Low Shear 30 sinus” (Contraves, Switzerland). All measurements were carried out by a MS 1/1 measuring system at temperature (T = 37 °C ± 0,1 °C). The test substance was located in the gap between the rotating measuring cup and the stationary measuring bob. A Low Shear 30 sinus can shifted a rigidly mounted additional driving device (oscillation drive) from oscillation to rotation. While measurements took place the test substance was subjected to an exactly defined frequency, thus providing forced oscillations.

Apparent steady shear viscosity was determined starting from a shear rate of 128.5 s⁻¹ and decreasing the rate in 11 steps. Complex dynamic viscosity was determined by measuring the angular displacement $\delta$ varying frequency from 0,000222 Hz up to 1,63 Hz at a firmly set amplitude of ± 30°. Low Shear 30 sinus allowed to register loss angle $\delta$ and torque $M_\text{rE}$, whereas $M_r$ (or viscosity $\eta_r$ respectively) and $M_E$ (or storage modulus $G'$ respectively) were calculated. Sinusoidal time-varying flow provided a basis for clear differentiation of the elastic and viscous properties of the material and understanding the role of viscoelasticity in a more complex time-varying flow, such as pulsatile flow.
Blood samples were provided by the National Center for Transfusion and Hematology and by the Department of Neurology, Medical University in Sofia.

2.2. Theoretical method

The generalized Maxwell-Gurevich-Rabinovich (MGR) model for polymers and metals, presented by Rabinovich in [1] is used herein to describe the non-Newtonian rheological properties of human blood, observed in our experiments. The MGR model has been deduced from the thermo-fluctuation theory. According to this model the rate of the full deformation at shear is equal to:

\[
\dot{\gamma} = \gamma_e + \dot{\gamma} + \gamma
\]

where \(\dot{\gamma}\) are shear rates or full strain rate;

\(\gamma_e\) is the elastic strain rate;

\(\dot{\gamma}\) is the residual strain rate;

\(\gamma^*\) is the high elastic strain rate

and

\[
\dot{\gamma}_e = \frac{\tau}{G}; \quad \dot{\gamma} = \frac{d\tau}{dt}
\]

where

\(G\) is the shear elastic modulus and

\[
\gamma^0 = \eta_0^0 \exp \left( -\frac{\tau}{m_G^0} \right)
\]

\(\eta_0^0\) is the initial coefficient of viscosity for the residual strain;

\(m_G^0\) is the logarithmic modulus of the residual shear strain rate;

\[
\gamma^* = \eta_0^* \exp \left( -\frac{\tau^*}{m_G^*} \right)
\]
where 
\( \eta_0^* \) is the initial viscosity coefficient for the high elastic strain;
\( m_G^* \) is the logarithmic modulus for the high elastic shear strain rate;
\( f_G^* \) is a function of the shear elastic modulus \( G \), the equilibrium shear modulus of the high elastic strain \( G_e \), shear stresses and strains, determined by:

\[
f_G^* = \left(1 + \frac{G_e}{G}\right) \tau - G_e \gamma
\]  

(5)

2.3. For the regime of steady shear and non-steady time dependent viscometric flow (\( \gamma = \text{const} \)) we consider two shear rate regions: a) \( \gamma < 0.03 \text{ s}^{-1} \) and b) \( \gamma > 0.03 \text{ s}^{-1} \). In the case of very low shear rates or region a) \( \gamma < 0.03 \text{ s}^{-1} \), it follows from our experiments that the full strain rate \( \dot{\gamma} \) consists of elastic and the high elastic strain rate, having the form:

\[
\dot{\gamma} = \dot{\gamma}_e + \dot{\gamma}
\]

(6)

or

\[
\frac{d\gamma}{dt} = \frac{d\tau}{dt} \frac{1}{G} + \eta_0^* \exp\left(-\frac{f_G^*}{m_G^*}\right)
\]

(7)

When \( \gamma = \text{const} \) and \( \dot{\gamma} = \frac{d\gamma}{dt} \), the equation for the shear stress rate has the form:

\[
\frac{d\tau}{dt} = G \gamma \left(1 - \frac{1}{1 + \frac{G_e}{G_e}} \frac{\Psi(\zeta^*)}{\Psi(\zeta_{\text{max}}^*)}\right)
\]

(8)

where \( \zeta^* = \frac{f_G^*}{m_G^*} \), \( \zeta_{\text{max}}^* = \frac{f_{G_{\text{max}}}^*}{m_G^*} \) and \( f_{G_{\text{max}}}^* \) is the maximum of \( f_G^* \) from (5) and it is found from experimental data. To facilitate parameter calculation when performing integration by means of the finite difference method, we introduce functions:

\[
\Psi(\zeta^*) = \zeta^* \exp(\zeta^*), \quad \Psi(\zeta_{\text{max}}^*) = \zeta_{\text{max}}^* \exp(\zeta_{\text{max}}^*)
\]

The initial conditions in this case are:

\[
t = 0, \gamma = 0, \tau = 0, f_G^* = 0
\]

(9)
Taking into account that $\gamma = \text{const}$ and $\gamma = \dot{\gamma} t$, the equation for $\tau$ takes the form:

$$\frac{d\tau}{d\gamma} = G \left( 1 - \frac{f_{G}^{*} \cdot \dot{\gamma}}{\eta_{0}} \exp \left( \frac{f_{G}^{*}}{m_{G}} \right) \right)$$

or it can be written in the form:

$$\frac{d\tau}{d\gamma} = G \left( 1 - \frac{1}{1 + \frac{G_{v}}{G_{\max}}} \frac{\Psi(\zeta^{*})}{\Psi(\zeta^{\max})} \right)$$

with initial conditions:

$$t = 0, \gamma = 0, \tau = 0, f_{G}^{*} = 0$$

It follows from the experimental data, observed at shear rates $\dot{\gamma} \geq 0.0237 \text{ s}^{-1}$ or in case b), that the high elastic strain $\gamma^{*} = 0$ and the full strain rate are determined by the elastic rate and the residual strain. In this case $G_{v} = 0$ and $f_{G}^{*} = f_{G}^{0} = 0$ and equation (1) takes the form:

$$\dot{\gamma} = \gamma^{*} + \gamma$$

Then

$$\frac{d\tau}{dt} = 1 \frac{d\tau}{dt} = G \left( 1 - \frac{\Psi(\zeta^{0})}{\Psi(\zeta^{\max})} \right)$$

or

$$\frac{d\tau}{dt} = G \left( 1 - \frac{\Psi(\zeta^{0})}{\Psi(\zeta^{\max})} \right),$$

where: $\zeta^{0} = \frac{\tau}{m_{G}^{0}}$, $\zeta^{\max} = \frac{\tau}{m_{G}^{0}}$ and $\Psi(\zeta^{0}) = \zeta^{0} \exp(\zeta^{0})$, $\Psi(\zeta^{\max}) = \zeta^{\max} \exp(\zeta^{\max})$ are functions, introduced to facilitate parameter calculation when performing integration by means of the finite difference method.
\[ \frac{d \tau}{d \gamma} = G \left( 1 - \frac{\tau}{\eta_0} \exp \left( \frac{\tau}{m_0} \right) \right) \]  

or it is equal to:

\[ \frac{d \tau}{d \gamma} = G \left( 1 - \frac{\psi(\zeta_0)}{\psi(\zeta_{\text{max}})} \right) \]

When \( \frac{d \tau}{d \gamma} = 0 \), then the yield shear stress \( \tau_{\text{max}} \) is found from (17).

\[ \eta_0 \dot{\gamma} = f^\gamma_{\text{max}} \exp \left( \frac{f^\gamma_{\text{max}}}{m_G} \right) \]

\[ \eta_0 \dot{\gamma} = \tau_{\text{max}} \exp \left( \frac{\tau_{\text{max}}}{m_G} \right) \]

Using the MGR model we thus set forth relationship \((\tau-\gamma)\) via equations (12) and (17) and relationship \((\tau-t)\) via equations (8) and (19). They are integrated using the method of the finite differences to find the model coefficients from experimental data.

2.4. Modeling of the time dependent properties of blood

Experimental results reveal that blood exhibits time dependent rheological behavior. When \( \dot{\gamma} \leq 0.03 \text{s}^{-1} \) we consider and integrate equations (8) and (17) with initial conditions (9), whose finite difference form is:

\[ \tau_i = \tau_{i-1} + G \gamma \Delta t \left[ 1 - \frac{1}{1 + \frac{G_s}{G}} \frac{\psi(\zeta_{i-1})}{\psi(\zeta_{\text{max}})} \right] \]  

Where
\[ \zeta_{i+1}^* = \left( 1 + \frac{G_n}{G} \right) \tau_{i-1}^* - G_\gamma \gamma(i-1) \Delta t / m_G^* \]  

(21)

For area b) or shear rates (strain rate) \( \gamma \geq 0.03 \text{ s}^{-1} \) equation (15) has the following finite difference form:

\[ \tau_i = \tau_{i-1} + G \gamma \Delta t \left( 1 - \frac{\Psi(\dot{\gamma}^0)}{\Psi(\dot{\gamma}^0)} \right), \]  

(22)

\[ \zeta_{i-1} = \frac{\tau_{i-1}}{m_G^*} \]  

(23)

2.5. When \( m_G^* \to \infty \) MGR model (10) is reduced to the classical Maxwell equation (24):

\[ \frac{d\tau}{dy} = G \left( 1 - \frac{\tau}{\eta_0^* \gamma} \right) \]  

(24)

When the initial conditions are \( t = 0, \tau = \tau_0 \), then the solution of (24) has the form:

\[ \tau = \eta_0^* \gamma \left( 1 - \exp \left( - \frac{G}{\eta_0^* \gamma} \right) \right) + \tau_0 \exp \left( - \frac{G}{\eta_0^* \gamma} \right) \]  

(25)

Equation (25) is the linear case of the MGR model and it describes the relationship \( \tau = f(\gamma, \gamma^0, \eta_0^*) \). Shear stress-deformation relations \( (\tau, \gamma) \) experimentally found by Picart et al [4] and shown in Fig.3, are described by equation (25).

The linear MGR model is used to model stress relaxation after flow cessation to \( \gamma = 0 \) and at a rectangular change of shear rates in the area a) \( \gamma < 0.03 \text{ s}^{-1} \). Its form is:

\[ \tau = a + (\tau_0 - a) \exp(-pt) \]  

(26)

where

\[ a = \frac{G_n \gamma^0}{1 + \frac{G}{\eta_0^*}}; \quad p = \frac{G}{\eta_0^*} \left( 1 + \frac{G_n}{G} \right) \]  

(27)
In area b) at $\gamma > 0.03 \text{s}^{-1}$, the model for shear stress relaxation at the same regime of shear rates, after flow cessation to $\gamma = 0$, has the form:

$$\tau = \tau_0 \exp(-pt), p = \frac{G}{\eta_0^0}$$

(28)

### 2.6. Dependence of blood dynamic viscosity on shear rates

is modeled by the modified MGR model (19). The logarithmic strain modulus is constant $m_G^0 = \text{const}$ in materials with stable structure, opposite to materials with varying structure. In this case equation (19) takes the form:

$$\tau = m_G^0 \exp \left( \frac{\tau \gamma}{\eta_0^0} \right)$$

(29)

To account for the shear rate effect on the fluid internal structure, we introduce function

$$m_G^0 = m_G^0 + b \gamma + a \gamma^k$$

(30)

where $m_G^0, a, b, k$ are constants, found from the experimental data.

Experimental data found by means of steady, time dependent and dynamic measurements are assessed by the above presented MGR theory and equations for different shear flow regimes. General solutions $\tau(t)$ of the Maxwell-Gureich-Rabinovich type equation are discussed in the case of stress relaxation (after cessation of the steady flow) and stress formation (under constant shear rate), as well as in the case of hysteresis.

### 2.7. Computational methods

Data analysis was performed using the software packages Jandel Scientific and Sigma Plot and a rheological software specially developed for that purpose. The method of finite differences was used to integrate the equations modeling blood rheological properties.

### 3. Results

#### 3.1. Results of modeling the time-dependent properties of blood at rectangular change of shear rates

Experimentally established time dependent properties of blood at rectangular change of shear rates confirm the conclusion, that blood exhibits viscoelastic behavior, which is shown in Fig.1a. The
initial destroying and further pre-structuring of blood in “rouleaux” at shear rates up to $\gamma = 0.0237 \, \text{s}^{-1}$ causes shear stress increase in time, which reveals elastic behaviour of the blood sample. The process of destroying and creating new connections is not well balanced. It leads to increase of the size of the related structural units, corresponding to a bigger resistance to flow at very low shear rates. Comparison of the experimental with theoretical data obtained by using the modified MGR model (20)-(21) is shown in Fig.1a. The coefficients of equation (20), found from stress development experimental data (Fig.1a) are: $m_0 = 31,292 \, \text{mPa}$, $f_G = 0.728 \, \text{mPa}$, $G = 8.12 \, \text{mPa}$, $G_\infty = 0.188 \, \text{mPa}$, $\zeta_{\max} = 0.0232$ and $\Delta t = 2.53164 \, \text{s}$.

Stress relaxation experimental data, shown in the same figure, are fitted by equations (26) and (27). The deviation of the theoretical approximation from the experimental data is up to 25% at large relaxation times, but close to 0% at small relaxation times. The coefficients of the model (26)-(27), derived from the experimental data are: $\eta_0 = 19,763 \, \text{mPa.s}$, $\gamma_c = 0.766$ (deformation, at which flow has been terminated), $G = 8.12 \, \text{mPa}$, $G_\infty = 0.157 \, \text{mPa}$. Modeling of blood thixotropic properties at a rectangular change of shear rates is described in detail and presented in [3].

**Fig.1a.** Stress growth (from $\gamma = 0$, to $\gamma = 0.0237 \, \text{s}^{-1}$) modeled by eq. (20) and stress relaxation - by (26)-(27) for the whole human blood (DA), $H=43\%$, $T=37^\circ \text{C}$; experimental points - $\mathbf{X}$, theoretical lines - $\mathbf{M}$. From [3]

**Fig.1b.** Stress growth (from $\gamma = 0$, to $\gamma = 0.0596 \, \text{s}^{-1}$) modeled by eqs. (22)-(23) and stress relaxation - by eqs. (26)-(27) for the whole human blood (DA), $H=43\%$, $T=37^\circ \text{C}$; experimental points - $\mathbf{X}$, theoretical lines - $\mathbf{M}$. From [3]

**Fig.1c.** Stress growth modeled by eqs. (22)-(23) (from $\gamma = 0$, to $\gamma = 1.285 \, \text{s}^{-1}$ and to $\gamma = 1.285 \, \text{s}^{-1}$) and stress relaxation - by eq. (28) for the whole human blood (DA), $H=43\%$, $T=37^\circ \text{C}$; exp. points - $\mathbf{X}$, theor. lines- $\mathbf{M}$, $\Delta \mathbf{M}$. From [3]

Experimental shear stress- time dependence at rectangular change of shear rate up to $\gamma = 0.0596 \, \text{s}^{-1}$ is shown in Fig.1b. It may be concluded from the experiment, that a part of RBCs are already arranged in structures or “rouleaux”, while another part of them are flowing as single cells. These structures form the “plateau” of the $(\tau-t)$ curve in Fig. 1b. When shear rates are higher, equal to 0, 0596 s$^{-1}$ (Fig.1b), 1,285 s$^{-1}$ and 5, 96 s$^{-1}$ (Fig.1c), stress development (under constant shear rate) is modeled by equations (22)-(23).
The higher shear rate leads to decrease of the size of the related structural units, corresponding to a less resistance to flow. The last two (t-τ) rheograms at shear rates of 1,285 s\(^{-1}\) and 5.96 s\(^{-1}\) in Fig.1c reveal the well-known “stress overshoot” behaviour of the human blood. It is seen that the MGR model describes very well the experimentally observed time-dependent properties. Stress relaxation described by the MGR model is shown in the same figures. It is seen in Fig.1a and Fig.1b, that stress relaxation after flow cessation has not been completed for the time of the experiment performance (about 10 s).

Experimental results for time dependent properties of blood and their modeling with the proposed MGR model (22)-(23) at shear rates of 1,285 s\(^{-1}\) and 5.96 s\(^{-1}\) are shown in Fig. 1c. “Stress overshoot” is observed after which blood is flowing. Following the experimental observations we conclude that blood exhibits elastic and plastic (residual) deformation. The MGR model in this case is based on the proposal, that the full strain rate consists of the elastic strain rate \(\gamma_e\) and the residual strain rate \(\gamma^0\). Comparison shows that the MGR model - equations (22) and (23), adequately describes this flow mechanism. It is seen in the same figure that the stress relaxation after cessation of flow (Fig. 1c), modeled by (28) at this shear rate, describes adequately the process. In this case when flow is terminated, particles connect to each other and the process of destroying and creating new connections is well balanced.

Parameters of model (22)-(23) calculated from the experimental data and coefficients, describing stress development at shear rates 0.0596 s\(^{-1}\), 1.285 s\(^{-1}\) and 5.96 s\(^{-1}\) - Fig. 1b and Fig.1c, are given in Table 1a. In the case of stress relaxation (after the cessation of a steady flow starting from shear rates 0.0596 s\(^{-1}\), 1.285 s\(^{-1}\) and 5.96 s\(^{-1}\) to 0 s\(^{-1}\)) equation (28) describes the observed phenomena. Comparison of experimental points and theoretical lines are shown in Fig. 1b and Fig. 1c. The coefficients of model (28) for the whole human blood are given in Table 1b.

Table 1a.
Numerical values of the parameters of the equations (22)-(23), modeling stress development at rectangular change of shear rates from 0 to 0.0596 s\(^{-1}\), 1.285 s\(^{-1}\) and 5.96 s\(^{-1}\) for the whole human blood (DA), H=43%, T=37°C.

<table>
<thead>
<tr>
<th>Shear rates</th>
<th>(\zeta_{max}^0)</th>
<th>(m_G^0) [mPa.s]</th>
<th>(\tau_{max}^0) [mPa.s]</th>
<th>(\Psi(\zeta_{max}^0))</th>
<th>(\Delta t) [s]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0596 s(^{-1})</td>
<td>0.06736</td>
<td>31.92</td>
<td>2.1080</td>
<td>0.07205</td>
<td>2</td>
</tr>
<tr>
<td>1.285 s(^{-1})</td>
<td>0.57791</td>
<td>31.292</td>
<td>18.084</td>
<td>1.03</td>
<td>1</td>
</tr>
<tr>
<td>5.96 s(^{-1})</td>
<td>1.63557</td>
<td>31.292</td>
<td>51.292</td>
<td>8.39438</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 1b.
Numerical values of the parameters of equations (28), modeling stress relaxation after cessation of the rectangular flow, starting from different shear rates - 0.0596 s\(^{-1}\), 1.285 s\(^{-1}\) and 5.96 s\(^{-1}\), for the whole human blood (DA), H=43%, T=37°C.

<table>
<thead>
<tr>
<th>Shear rates</th>
<th>(\tau_0) [mPa]</th>
<th>(G/\eta^0) [s(^{-1})]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0596 s(^{-1})</td>
<td>1.9854</td>
<td>0.3422</td>
</tr>
<tr>
<td>1.285 s(^{-1})</td>
<td>18.084</td>
<td>0.78469</td>
</tr>
<tr>
<td>5.96 s(^{-1})</td>
<td>57.1803</td>
<td>0.9438</td>
</tr>
</tbody>
</table>

The progressive breakdown and formation of rouleaux under shear rates is proposed to be one of the major causes of blood thixotropic behavior. The increase in shear rate leads to a transient reduction in size and number of these associated structural units, corresponding to a smaller resistance against flow. At
higher shear rates $\dot{\gamma} = 1.285 \ s^{-1}$ and 5, 96 s$^{-1}$ RBC aggregates can not form larger structures and fast flowing is observed. In those cases shear stresses reach a full relaxation and only elastic deformation is revealed. This behavior is modeled by equation (28). The proposed MGR model does not describe the observed “stress overshoot” in this form. For that purpose, the model can be completed including the energy of activation of RBC aggregates due to the change of their structure.

3.2. Results of the modeling blood time-dependent properties at triangular change of shear rates

Experimentally found hysteresis for normal conserved human blood (300 ml blood/75 ml L-12), H=38%, T=37°C at triangular change of shear rates $\dot{\gamma} = \alpha t$ for $0 < t < t_{\text{max}}$ - loading and $\dot{\gamma} = \alpha (2t_{\text{max}} - t)$ for $t_{\text{max}} < t < 2t_{\text{max}}$ – discharge is shown in Fig. 2. The form of the hysteresis loops depends on the applied shear rates and on the duration of shear – Fig. 2: a) $\dot{\gamma}_{\text{max}} = 0.0596 \ s^{-1}$ and $t_{\text{max}}=30$ s; b) $\dot{\gamma}_{\text{max}} = 1.18 \ s^{-1}$ and $t_{\text{max}}=30$ s. The observed experimental relationships $(\tau - \gamma)$ under triangular change of shear rates a) and b) are modeled by equation (19). The MGR model has been modified taking into account shear rate effect on structuring applied through the logarithmic modulus of the residual strain rate at shear $m_G^0$. This modulus has the form of relation (30), while at relaxation – that of eq. (31):

$$m_G^0 = m_G^0 + b \gamma + a \gamma^2 + c \gamma^3$$

(31)

The numerical values of the model equation parameters found from the experiments, are shown in Table 2.

![Blood hysteresis loops](image)

Fig. 2. Blood hysteresis loops at triangular change of shear rates, modeled by the modified MGR model (19) a) $\dot{\gamma}_{\text{max}} = 0.0596 \ s^{-1}$ and $t_{\text{max}}=30$ s; b) $\dot{\gamma}_{\text{max}} = 1.18 \ s^{-1}$ and $t_{\text{max}}=30$ s. for normal human conserved blood (300 ml blood/75 ml L-12), H=38%, T=37°C, LS30 Contraves
Experimentally determined hysteresis loops of normal conserved human blood under triangular change of shear rates and the model results, found using the MGR model modification (19), are shown in Fig. 2. The model describes very well the observed loading and discharge curves and the hysteresis loop, except for the special experimental data. The comparison shows that equation (19) can be used to calculate the hysteresis loops.

Table 2.
Numerical values of the parameters of the model equation (19) for hysteresis loop for normal human blood conserved by L-12 (300 ml blood / 75 ml L-12), H = 38 %, T = 37 °C.

<table>
<thead>
<tr>
<th>Parameters of eq. (19)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Models</td>
</tr>
<tr>
<td>( M_i ), (19), (30) ( \eta_0^0 = 0.0596 \text{ s}^{-1} )</td>
</tr>
<tr>
<td>( M_d ), (19), (31) ( \eta_0^0 = 0.0596 \text{ s}^{-1} )</td>
</tr>
<tr>
<td>( M_i ), (19), (30) ( \eta_0^0 = 1,18 \text{ s}^{-1} )</td>
</tr>
<tr>
<td>( M_d ), (19), (31) ( \eta_0^0 = 1,18 \text{ s}^{-1} )</td>
</tr>
</tbody>
</table>

To verify model (25), the experimental results found by Picart et al. [4] were used to describe the shear stress-deformation (product of time and shear rate) relationship for a human blood sample, H=70%, under different shear rates - \( 10^{-3} \text{ s}^{-1} \), \( 3.10^{-3} \text{ s}^{-1} \), \( 10^{-2} \text{ s}^{-1} \) and \( 3 \cdot 10^{-2} \text{ s}^{-1} \), LS40, T=25°C. The comparison is shown in Fig. 3. It is seen in the figure that the relationship shear stresses-strain \( (\tau - \gamma) \) is very well described by the linear solution of the MGR model (24)-(25). The numerical values of the parameters of equations (24) and (25) calculated from experimental data are shown in Table 3.

Table 3.

<table>
<thead>
<tr>
<th>Parameters of model eqs. (24)-(25)</th>
</tr>
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<tbody>
<tr>
<td>( \gamma ), [s^{-1}]</td>
</tr>
<tr>
<td>( 10^{-3} )</td>
</tr>
<tr>
<td>( 3 \cdot 10^{-3} )</td>
</tr>
<tr>
<td>( 10^{-2} )</td>
</tr>
<tr>
<td>( 3 \cdot 10^{-2} )</td>
</tr>
</tbody>
</table>
Fig. 3. Modeling the relationship $\tau - \gamma$ by means of eq. (25) for a human blood sample, $H=70\%$, under different shear rates $-10^{-3} \text{s}^{-1}$, $3 \cdot 10^{-3} \text{s}^{-1}$, $10^{-2} \text{s}^{-1}$ and $3 \cdot 10^{-2} \text{s}^{-1}$, LS40, $T=25^\circ \text{C}$ (from Picart et al.[4]). Experimental data: $\tau_{\exp 1} (10^{-3} \text{s}^{-1})$; $\tau_{\exp 2} (3 \cdot 10^{-3} \text{s}^{-1})$; $\tau_{\exp 3} (10^{-2} \text{s}^{-1})$; $\tau_{\exp 4} (3 \cdot 10^{-2} \text{s}^{-1})$; Theoretical lines: $\tau_{M1} (10^{-3} \text{s}^{-1})$; $\tau_{M2} (3 \cdot 10^{-3} \text{s}^{-1})$; $\tau_{M3} (10^{-2} \text{s}^{-1})$; $\tau_{M4} (3 \cdot 10^{-2} \text{s}^{-1})$.

3.3. Results of modeling the rheological properties of blood under steady shear

The apparent dependence of viscosity on shear rates under steady shear flow ($\eta - \gamma$) is modeled by equation (32):

$$\eta^0 \exp \frac{\gamma \eta^0}{m_G^0} = \eta_0^0,$$

(32)

Shear rate effect on the structure formation is presented by eq. (30) $m_G^0 = m_0^0 + b \gamma + a \gamma^k$, where $m_0^0, a, b, k$ are constants found from the experimental data and given in Table. 4. The relationships $(\tau - \dot{\gamma})$ under steady flow is modeled by equation (19).
Fig. 4. Shear stress as a function of shear rates (19) and apparent viscosity as a function of shear rates (32) for normal human blood samples with different hematocrits, respectively: experimental points (exp) and theoretical model lines (m1 or m2) for:

- \( \tau, \eta \) \( \text{exp}(H=30\%) \); 
- \( \tau, \eta \) \( \text{m1}(H=30\%) \); 
- \( \tau, \eta \) \( \text{m2}(H=30\%) \);
- \( \tau, \eta \) \( \text{exp}(H=40\%) \); 
- \( \tau, \eta \) \( \text{m1}(H=40\%) \); 
- \( \tau, \eta \) \( \text{m2}(H=40\%) \);
- \( \tau, \eta \) \( \text{exp}(H=46\%) \); 
- \( \tau, \eta \) \( \text{m1}(H=46\%) \); 
- \( \tau, \eta \) \( \text{m2}(H=46\%) \);
- \( \tau, \eta \) \( \text{exp}(H=50\%) \); 
- \( \tau, \eta \) \( \text{m1}(H=50\%) \); 
- \( \tau, \eta \) \( \text{m2}(H=50\%) \);
- \( \tau, \eta \) \( \text{exp}(H=60\%) \); 
- \( \tau, \eta \) \( \text{m1}(H=60\%) \); 
- \( \tau, \eta \) \( \text{m2}(H=60\%) \);
- \( \tau, \eta \) \( \text{exp}(H=70\%) \); 
- \( \tau, \eta \) \( \text{m1}(H=70\%) \); 
- \( \tau, \eta \) \( \text{m2}(H=70\%) \).

Table 4. Numerical values of the parameters of the modified Maxwell-Gurevitch-Rabinovich model (19) for the whole normal human blood with different hematocrits under steady shear at 37°C, LS 30 Contraves

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Hematocrit H, %</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>30</td>
</tr>
<tr>
<td>( \eta_0, [\text{mPa.s}] )</td>
<td>30</td>
</tr>
<tr>
<td>( m_0, \eta_0 [\text{mPa}] )</td>
<td>2,24</td>
</tr>
<tr>
<td>( b )</td>
<td>0,359</td>
</tr>
<tr>
<td>( a )</td>
<td>3,2575</td>
</tr>
<tr>
<td>( k )</td>
<td>0,75</td>
</tr>
</tbody>
</table>

3.4. Results of the modeling blood viscoelastic properties by means of the modified MGR model

The experimental dependence of the viscous part of the complex viscosity of human blood of a patient with CVD (H=49%, T=37°C) on the oscillatory shear rate, and the theoretical curves found using model equations (33) and (34), are shown in Fig.5. Dynamic viscosity - oscillatory shear rate dependence
of blood is described by the linear MGR model (33) and by the modified MGR model (34). The modified model describes very well changes of the dynamic viscosity, especially within the range \(2 \text{ s}^{-1} \leq \dot{\gamma} \leq 20 \text{ s}^{-1}\). It is seen in the same figure that the experimental data can be presented as separate linear parts with different parameters \(a\) and \(n\), using the linear model (33). The numerical values of the parameters of eqs. (33) and (34), derived from the experimental oscillatory shear rate dependence of the viscous part of the complex viscosity of blood of a patient with CVD are shown in Table 5.

Linear case - Model 1, equation (33):

\[
\eta^0 = \lg a - n \lg \dot{\gamma} \tag{33}
\]

Nonlinear case - Model 2, equation (34):

\[
\eta^0 \exp \left[ \frac{1 + n}{m_0^0} \dot{\gamma}^n \right] = \eta_0^0 \tag{34}
\]

Fig.5. The oscillatory experimental shear rate dependence of viscous part of the complex viscosity of human blood of a patient with CVD \(\times \eta_{\text{exp}}^{H=49\%} \), \(T=37^\circ\text{C}\), LS30 sinus and theoretical lines \(\Delta \eta_{\text{M1}}^{H=49\%} \), \(\star \eta_{\text{M2}}^{H=49\%}\) found by modeling the dynamic viscosity of blood employing the MGR model (33) and (34) respectively.

Table 5.

<table>
<thead>
<tr>
<th>Parameters of the eqs. (33) and (34)</th>
<th>0.11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1: (26.7 \leq \dot{\gamma} \geq 49.3)</td>
<td>46,03 (10^{10})</td>
</tr>
<tr>
<td>Model 2</td>
<td>73,97 (10^{11})</td>
</tr>
<tr>
<td>Model 1: (0.267 \leq \dot{\gamma} \geq 3.11)</td>
<td>2,16</td>
</tr>
<tr>
<td>Model 1: (3.11 \leq \dot{\gamma} \geq 26.7)</td>
<td>8,66</td>
</tr>
</tbody>
</table>
5. Discussion

The experimental relationships provided by the steady, time dependent and dynamic rheological measurements were modeled by the discussed MGR theory and by equations for different shear flow regimes, whereas the latter were experimentally realized. For the regime of steady shear and non-steady time dependent viscometric flow ($\gamma = \text{const}$) we considered two shear rate areas: a) $\gamma < 0.03 \text{ s}^{-1}$ and b) $\gamma > 0.03 \text{ s}^{-1}$. In case a) stress development revealed elastoplastic behaviour, while in case b) – viscoelastic one. Stress relaxation in case a) was not completed, while in case b) shear stresses attained full relaxation and elastic deformation was revealed, only. The observed experimental relationships ($\tau(\gamma)$ under triangular change of shear rates a) and b) were modeled by equation (19), which is an exponential general solution $\tau(t)$ of the Maxwell-Gureich-Rabinovich type equation in the case of hysteresis. The MGR model was modified taking into account the shear rates effect on the structuring applied via the logarithmic modulus of the rate of residual strain at shear $m_G^0$.

Blood steady flow experimental relationships ($\tau - \gamma$) were modeled by the same equation (19). The corresponding ($\eta - \gamma$) relationships for different hematocrits were modeled by equation (32). Increase of the initial coefficient of viscosity for the residual strain $\eta_0^0$ and the logarithmic modulus of the rate of residual strain at shear $m_G^0$, with increasing the hematocrit, presented in Table 4, was observed. Dynamic viscosity - oscillatory shear rate dependence for blood was described by the linear MGR model (33) and the modified MGR model (34). The modified model describes very well changes of the dynamic viscosity, divided in three separate linear parts, especially within the range $2 \text{ s}^{-1} \leq \gamma \leq 20 \text{ s}^{-1}$. The elastic modulus of shear $G$; the initial coefficient of viscosity $\eta_0$ and the logarithmic modulus of the velocity of the deformation $m$ provide a functional relation between $\left(\tau, \eta, \gamma, \gamma^*\right)$ and characterize the rheological behaviour of blood at the considered flow regimes. For accurate description of the observed relationships some modifications of the MGR model were proposed, introducing nonlinear dependences of the logarithmic deformation modulus $m$ on shear rates $\gamma$. The parameters of the proposed models were found from experimental data.

Very similar molecular or microscopic processes associated with viscoelasticity and thixotropy provide the basis of a unifying description of these diverse rheological properties. Generally these properties are connected with the effects of the fluid inner structure on the rheological behaviour. The change of blood flow in time is typical for viscoelastic and thixotropic fluids and this phenomenon is related to the influence of the inner structure on blood flow. In the case of thixotropy when flow is stopped, particles connect with each other and the process of destroying and creating new connections is well balanced. Connection forces are weak. The higher shear rate leads to decrease of the size of the related structural units, corresponding to a less resistance to flow. Similarly, the macromolecules of the viscoelastic fluid form a network of temporal links, and the destruction and creation of links is balanced.
When we maintained larger fluid shear rates, more macromolecules were destroyed for a unit time. Consequently, thixotropy and viscoelasticity were related to similar time effects. In this sense, appropriate theories were developed, describing the mechanisms of thixotropy and viscoelasticity - see G. Thurston [5], P. Riha [6] and D. Quemada [7]. The presented MGR theory and the proposed equations describe in detail the rheological behaviour and characteristics of blood at different regimes of viscometric shear flow.

6. Conclusion

The modified model of A.L.Rabinovich or Maxwell-Gurevich-Rabinovich (MGR) model was chosen to describe the experimentally observed rheological properties of blood under steady and non-steady flow conditions. The MGR model enables one to investigate changes of the microstructure of blood undergoing deformation through blood parameters found from experimental data. By means of the Low Shear 30 sin oscillation rheometer, shear stresses, viscosity $\eta$ and storage modulus $G'\tilde{\eta}$ were determined, varying frequency and shear rates as ruling parameters. Rheograms of change of viscosity ($\eta$), respectively shear stresses ($\tau$) after sudden step increase of shear rates ($\gamma$) from 0 to 0.0237 s$^{-1}$; 0.0596 s$^{-1}$; 1.285 s$^{-1}$ and 5.96 s$^{-1}$ as well as under triangular change of shear rates, are plotted. Model equations were proposed through MGR model modifications, introducing nonlinear dependences of the logarithmic deformation modulus $m$ on shear rates $\gamma$. General solutions of the Maxwell-Gurevich-Rabinovich type equation were discussed in the case of stress formation (under constant shear rate) and stress relaxation (after the cessation of the steady flow) at a rectangular and triangular change of shear rates and under steady and oscillation sinusoidal flow. The rheological parameters obtained from the modified model of Rabinovich describe the micromechanical rheological properties of human blood. The proposed equations can be successfully used to complete the system of equations for blood under different flow regimes.

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References